(Measurements and Instrumentation)

Second Year
2024 – 2025

Measurements and Instrumentation

Lecture 1

Sensors and Transducers

Measurement: The process of converting physical parameters into meaningful numerical values by transforming variables into corresponding analogs, such as pneumatic pressure, electrical voltage or current, or digitally encoded signals, in order to obtain information regarding the physical values of the variable.

Instrument: A device used to determine the value or magnitude of a quantity or variable, forming part of the collection of devices known as instrumentation, which are used in a measurement system.

Sensor: A device that performs the initial measurement and energy conversion of a variable into analogous digital, electrical, or pneumatic information. It is often referred to as a transducer, although the term "sensor" is preferred for the initial measurement device. A sensor converts physical variables such as pressure, temperature, and flow into an analog quantity, often electrical in nature.

Classification of Sensors

There are several classifications of sensors:

1. Active and Passive:
   Active Sensors are those which require an external excitation signal or a power signal.
   Passive Sensors, on the other hand, do not require any external power signal and directly generates output response.
2. Electric, Biological, Chemical, Radioactive etc. based on the means of detection used in the sensor.
3. Photoelectric, Thermoelectric, Electrochemical, Electromagnetic, Thermo optic, etc. based on conversion phenomenon i.e. the input and the output.
4. Analog and Digital Sensors.
   Analog Sensors produce an analog output i.e. a continuous output signal with respect to the quantity being measured.
   Digital Sensors, in contrast to Analog Sensors, work with discrete or digital data. The data in digital sensors, which is used for conversion and transmission, is digital in nature.

Different Types of Sensors

The following is a list of different types of sensors that are commonly used in various applications. All these sensors are used for measuring one of the physical properties like Temperature, Resistance, Capacitance, Conduction, Heat Transfer etc.

- Temperature Sensor
- Proximity Sensor
- Accelerometer
- IR Sensor (Infrared Sensor)
- Pressure Sensor
- Light Sensor
- Ultrasonic Sensor
- Flow and Level Sensor

etc.

Detailed Classification of Sensors for Manufacturing Applications

Sensors can be classified into various groups according to the factors such as measurand, application fields, conversion principle, energy domain of the measurand and thermodynamic considerations. These general classifications of sensors are well described in the references. So, Detail classification of sensors in view of their applications in manufacturing is as follows:

1. Displacement, position and proximity sensors
    - Potentiometer
    - Strain-gauged element
    - Capacitive element
    - Differential transformers
    - Eddy current proximity sensors
    - Inductive proximity switch
    - Optical encoders
    - Pneumatic sensors
    - Proximity switches (magnetic)
    - Hall effect sensors
2. Velocity and motion
    - Incremental encoder
    - Tachogenerator
    - Pyroelectric sensors

3. Force
    - Strain gauge load cell
4. Fluid pressure
    - Diaphragm pressure gauge
    - Capsules, bellows, pressure tubes
    - Piezoelectric sensors
    - Tactile sensor
5. Liquid flow
    - Orifice plate
    - Turbine meter
6. Liquid level
    - Floats
    - Differential pressure
7. Temperature
    - Bimetallic strips
    - Resistance temperature detectors
    - Thermistors
    - Thermo-diodes and transistors
    - Thermocouples
    - Light sensors
    - Photo diodes
    - Photo resistors
    - Photo transistor

Principle of operation of these transducers and their applications in manufacturing are presented in the next lectures.

Transducer: A device that converts variations of one variable into variations of another variable, such as converting current changes into voltage changes. For example, a transducer might convert a voltage into a proportional current. In other words, all sensors are transducers, but not all transducers are sensors.

Sensor/Transducers Specifications

There are a number of performance related parameters of a transducer or measurement system. These parameters are called as sensor specifications. Sensor specifications inform the user to the about deviations from the ideal behavior of the sensors. Following are some various specifications of a sensor/transducer system.

1. **Range**

region between the limits within which a variable is to be measured. Thus, "a temperature is to be measured in the range of 20°C to 250°C" defines the range. (See also Span).

2. **Span**

The algebraic difference between the upper-range value and the lower-range value. Thus, a temperature in the range of 20°C to 250°C has a span of 230°C. (See also Range).

3. **Error**

Error is the difference between the result of the measurement and the true value of the quantity being measured. A sensor might give a displacement reading of 29.8 mm, when the actual displacement had been 30 mm, then the error is –0.2 mm.

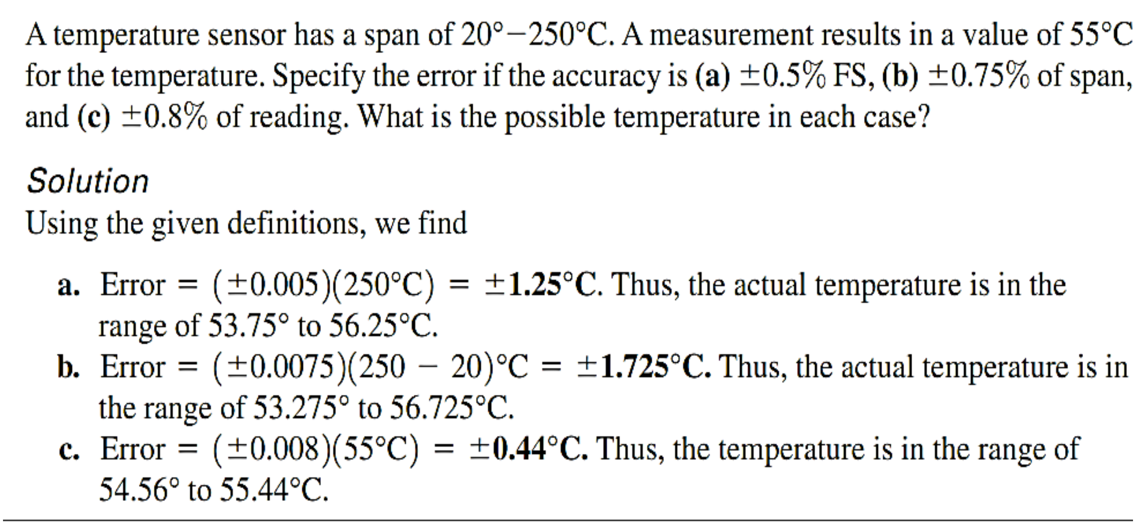
4. **Accuracy**

This term is used to specify the maximum overall error to be expected from a device, such as measurement of a variable.

Accuracy is usually expressed as the inaccuracy and can appear in several forms:

1. Measured variable; the accuracy is ± 2°C in some temperature measurement. Thus, there would be an uncertainty of ±2°C in any value of temperature measured.
2. Percentage of the instrument full-scale (FS) reading. Thus, an accuracy of ±0.5% FS in a 5-V full-scale range meter would mean the inaccuracy or uncertainty in any measurement is ±0.025 V.
3. Percentage of instrument span—that is, percentage of the range of instrument measurement capability. Thus, for a device measuring ±3% of span for a 20 to 50 psi range of pressure, the accuracy would be $(\pm 0.03)(50 - 20) = \pm 0.9$ psi.
4. Percentage of the actual reading. Thus, for a ±2% of reading voltmeter, we would have an inaccuracy of ±0.04 V for a reading of 2 V.

Example 1:



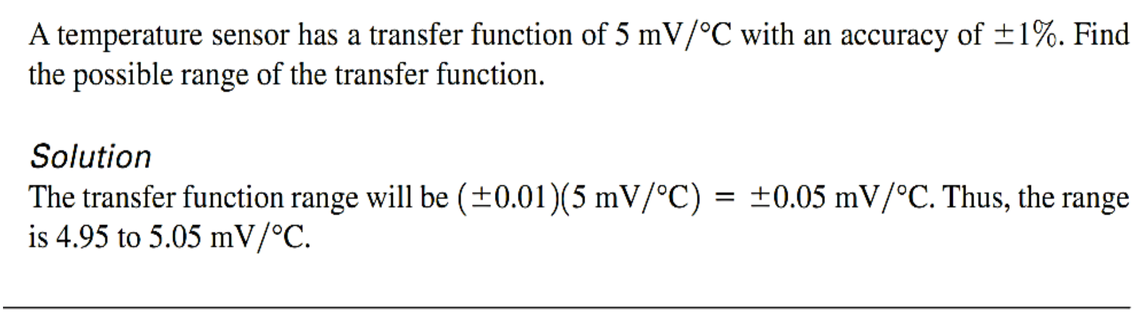
A temperature sensor has a span of 20°−250°C. A measurement results in a value of 55°C for the temperature. Specify the error if the accuracy is **(a)** ±0.5% FS, **(b)** ±0.75% of span, and **(c)** ±0.8% of reading. What is the possible temperature in each case?

Solution

Using the given definitions, we find

- **a.** Error = (±0.005)(250°C) = **±1.25°C**. Thus, the actual temperature is in the range of 53.75° to 56.25°C.
- **b.** Error = (±0.0075)(250 − 20)°C = **±1.725°C.** Thus, the actual temperature is in the range of 53.275° to 56.725°C.
- **c.** Error = (±0.008)(55°C) = **±0.44°C.** Thus, the temperature is in the range of 54.56° to 55.44°C.

Example 2:

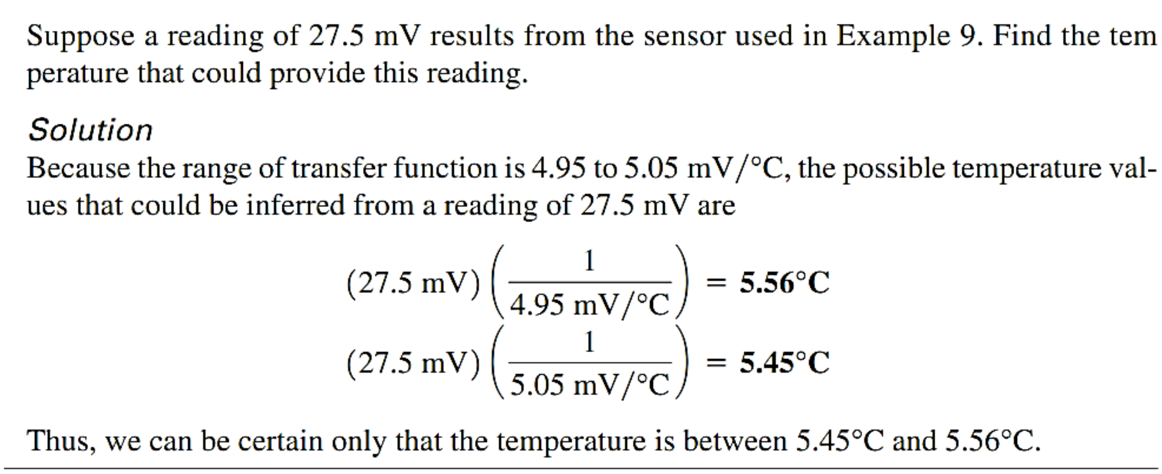


A temperature sensor has a transfer function of 5 mV/°C with an accuracy of ±1%. Find the possible range of the transfer function.

Solution

The transfer function range will be (±0.01)(5 mV/°C) = ±0.05 mV/°C. Thus, the range is 4.95 to 5.05 mV/°C.

Example 3:



Suppose a reading of 27.5 mV results from the sensor used in Example 9. Find the temperature that could provide this reading.

Solution

Because the range of transfer function is 4.95 to 5.05 mV/°C, the possible temperature values that could be inferred from a reading of 27.5 mV are

$$(27.5\ \text{mV})\left(\frac{1}{4.95\ \text{mV}/°\text{C}}\right) = \mathbf{5.56°C}$$

$$(27.5\ \text{mV})\left(\frac{1}{5.05\ \text{mV}/°\text{C}}\right) = \mathbf{5.45°C}$$

Thus, we can be certain only that the temperature is between 5.45°C and 5.56°C.

5. Sensitivity

The ratio of the change in output magnitude to the change of input magnitude, under steady-state conditions, for a measurement device.

6. Resolution

The minimum detectable change of some variable in a measurement system.

7. **Reliability:** Indicates a device's ability to perform its function without failure for a specified time, expressed in hours, years, or Mean Time Between Failures (MTBF). Based on accelerated lifetime testing, provided by the manufacturer.

8. **Stability**
   The sensor's ability to maintain consistent output for a constant input over time. "Drift" measures the change in output, expressed as a percentage of the full range.

Measurements and Instrumentation

Lecture 2

Strain Gauge

Strain Gauge

An electrical conductor whose resistance changes as it is strained.

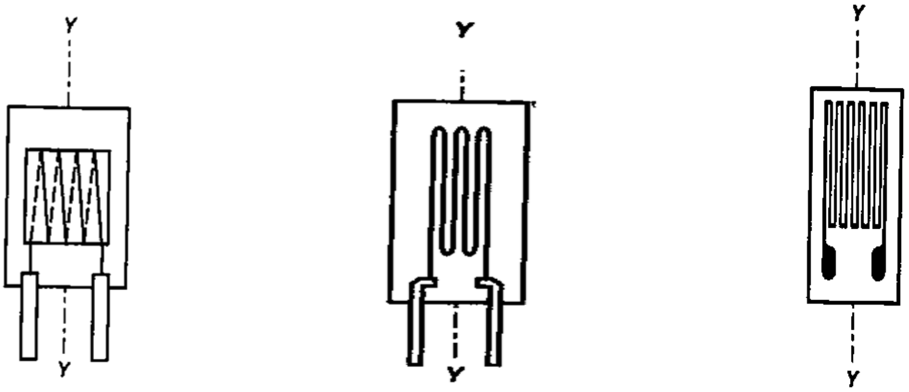
Applications of Strain Gauges:

1. Experimental Stress Analysis and Failure Analysis:
    - Multi-axial stress fatigue testing
    - Proof testing
    - Residual stress measurement
    - Vibration measurement
    - Torque measurement
    - Bending and deflection measurement
    - Compression and tension measurement
    - Strain measurement
2. Sensors in Various Fields:
    - Machine tools
    - Automotive safety
    - Hydraulic or pneumatic presses
    - Impact sensors in aerospace vehicles

Types of Strain Gauges:

1. Bonded Gauges:
    - Wrap-around Wire Strain Gauge: used for high resistance gauges (several hundred ohms).
    - Flat-grid Strain Gauge: Generally, has a resistance of not much more than one hundred ohms.
    - Etched Foil Strain Gauge: Similar to flat-grid strain gauges, with resistance not exceeding one hundred ohms.

These strain gauges must be fixed with a suitable adhesive to the surface they measure. Proper bonding embeds the electrical conductor in the bonding material, causing the resistance to increase under tensile strain and decrease under compressive strain in equal proportion, see Figure 2.1.



(1) (2) (3)

Figure 2.1 Strain gauge type: (1) Wrap-around (2) Flat-grid (3) Etched foil

2. Unbonded gauges

This type is used to give an electrical output signal proportional to a very small displacement of one body relative to another body.

One of it is application is an accelerometer, see Figure 2.2.

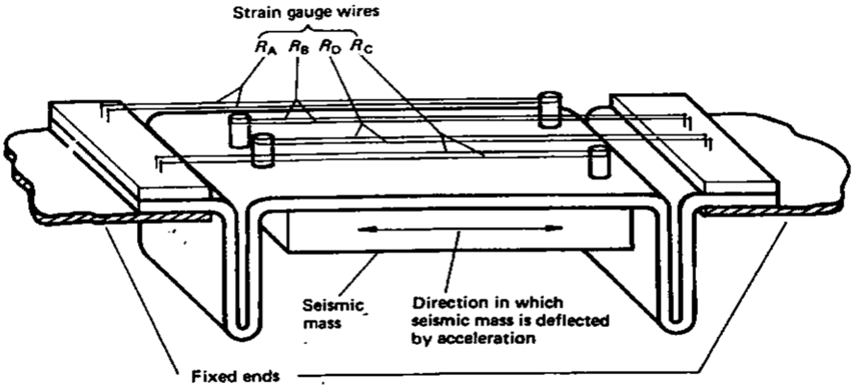


Figure 2.2 A strain gauge accelerometer

Strain Calculation

The relationship between strain and electrical resistance is given by the equation: $\frac{\delta R}{R} = k\varepsilon$

Where:

ε: is strain. That is, $\varepsilon = \frac{\delta l}{l} = \frac{elongation}{original\ length}$

R: is the original resistance of the strain gauge.

δR: is the change in resistance due to ε.

k: gauge factor (manufacturer determines this value).

Where, $\frac{\delta R}{R}$ is electrical strain and $\frac{\delta l}{l}$ is mechanical strain. From above equation it is clear that electrical strain is proportional to mechanical strain.

Bridge Circuit (Null Method)

Because of the actual change of strain gauge resistance is very small, Wheatstone bridge circuit has to be used.

R_A is the active gauge, bonded to the material whose strain is to be measured.

R_B is variable resistance.

R_C is a fixed resistance.

R_D is the dummy gauge, another strain gauge, identical to R_A but bonded to an unstrained piece of the same material as R_A is bonded to, and placed as close as possible to R_A. In order to cancel out any resistance change in R_A caused by change of temperature. To measure the resistance of R_A the bridge is balanced by adjusting R_B so that the voltages at P and Q are equal. Then:

$$R_A = R_B \times \frac{R_D}{R_C}$$

See Figure 1.3.

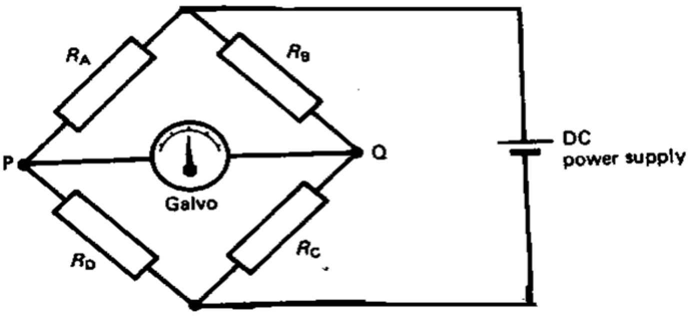


Figure 2.3 Wheatstone bridge

- The advantage of this method:

  The results are unaffected by variations in the voltage of the power supply. The problem, it takes the time and attention of an operator and transient changes of strain may pass unnoticed, see Figure 2.4.

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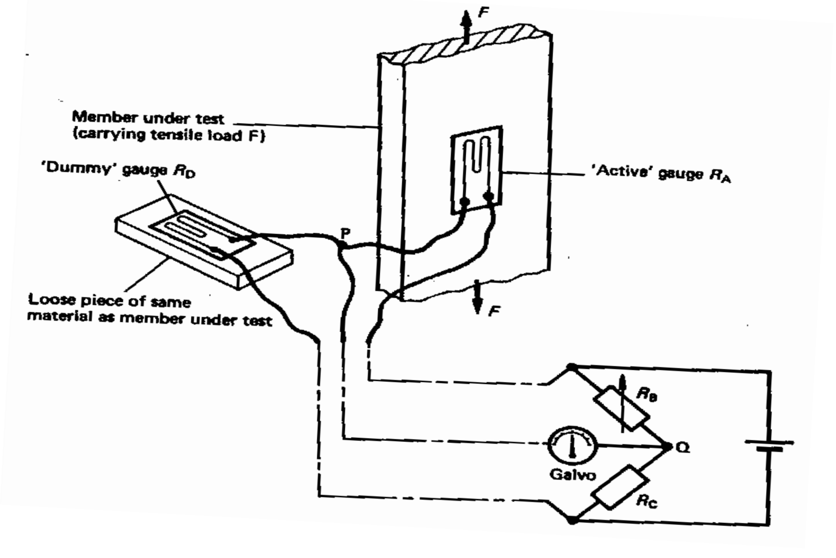


Figure 1.4 Wheatstone bridge as used for strain-gauging

Direct-reading bridge

In industry, strain gauges are often used to measure load rather than strain, providing a continuous output signal. The bridge is left unbalanced and calibrated with known loads, but the calibration is only accurate for one specific power supply voltage. Zero load can be adjusted with a low-resistance potentiometer, Rz., see Figure 2.6.

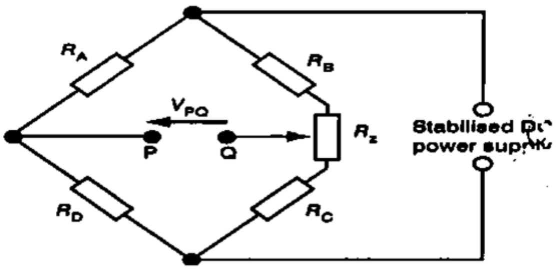


Figure 1.6 Potentiometer Rz enables bridge output voltage V_{PQ} to be zeroed

Semiconductor (piezo - resistive) strain gauges

It is made from crystalline materials in which large changes of electrical resistance occur when stress is applied.

➢ Advantage:

1) Large sensitivity compared with traditional strain gauge.
2) No need for amplification of their output.
3) Can be used to measure dynamic strains.

➢ Disadvantage:

The main problem their gauge factors may vary considerably as strain is applied, and also as temperature changes, so they are not quite as simple to use as ‘conventional’ gauges.

Possible Sources of Error in Strain Gauge Signals:

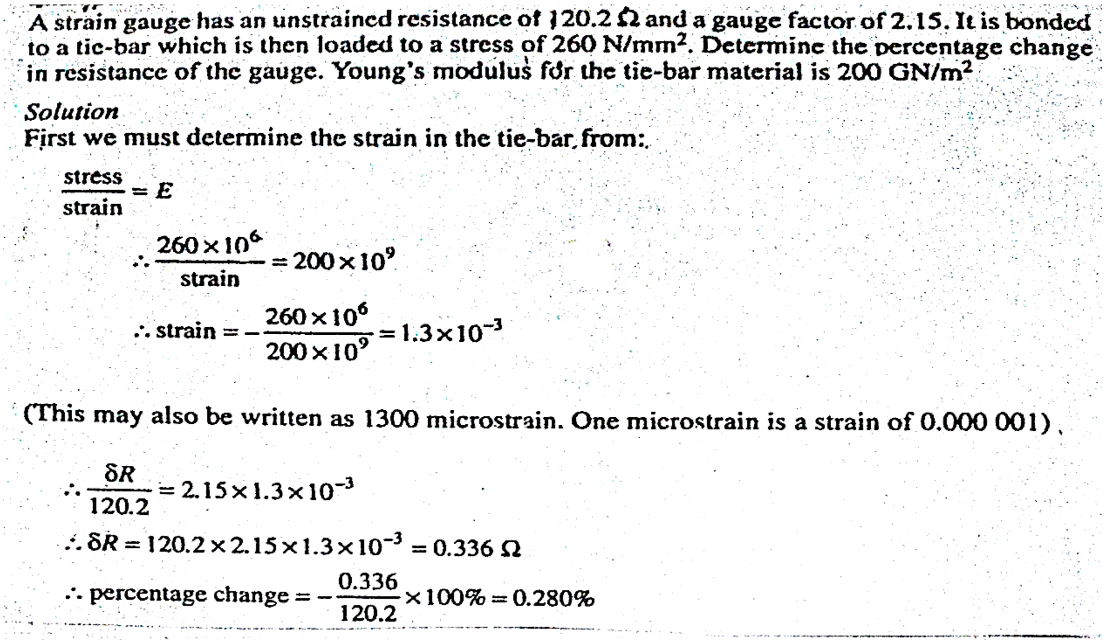
1. Cross-Sensitivity: Strain gauges respond to strain not only along their major axis but also to some extent at right angles due to the width of the resistance element, causing inaccuracies.
2. Bonding Faults: Imperfect bonding with adhesives can lead to creep, reducing the gauge factor and bridge output over time, especially under dynamic strain with many cycles.
3. Hysteresis: High strain can cause a permanent set in the gauge element, creating a hysteresis loop. Repeated loading/unloading cycles can narrow the loop, but severe hysteresis suggests faulty bonding.

4. Effects of Moisture: Absorption of water by the gauge or adhesive can

cause dimensional changes and false strain readings. Moisture can also create high resistance paths parallel to the gauge. Bonding in dry conditions or using water-repellent compounds can prevent this.

5. Temperature Change: Temperature variations, including heat generated by the gauge current, can affect readings. Allowing time for the temperature to stabilize after switching on the bridge is essential for accurate measurements.

Example 1:



A strain gauge has an unstrained resistance of 120.2 Ω and a gauge factor of 2.15. It is bonded to a tie-bar which is then loaded to a stress of 260 N/mm². Determine the percentage change in resistance of the gauge. Young's modulus for the tie-bar material is 200 GN/m²

Solution

First we must determine the strain in the tie-bar from:

$$\frac{\text{stress}}{\text{strain}} = E$$

$$\therefore \frac{260 \times 10^6}{\text{strain}} = 200 \times 10^9$$

$$\therefore \text{strain} = \frac{260 \times 10^6}{200 \times 10^9} = 1.3 \times 10^{-3}$$

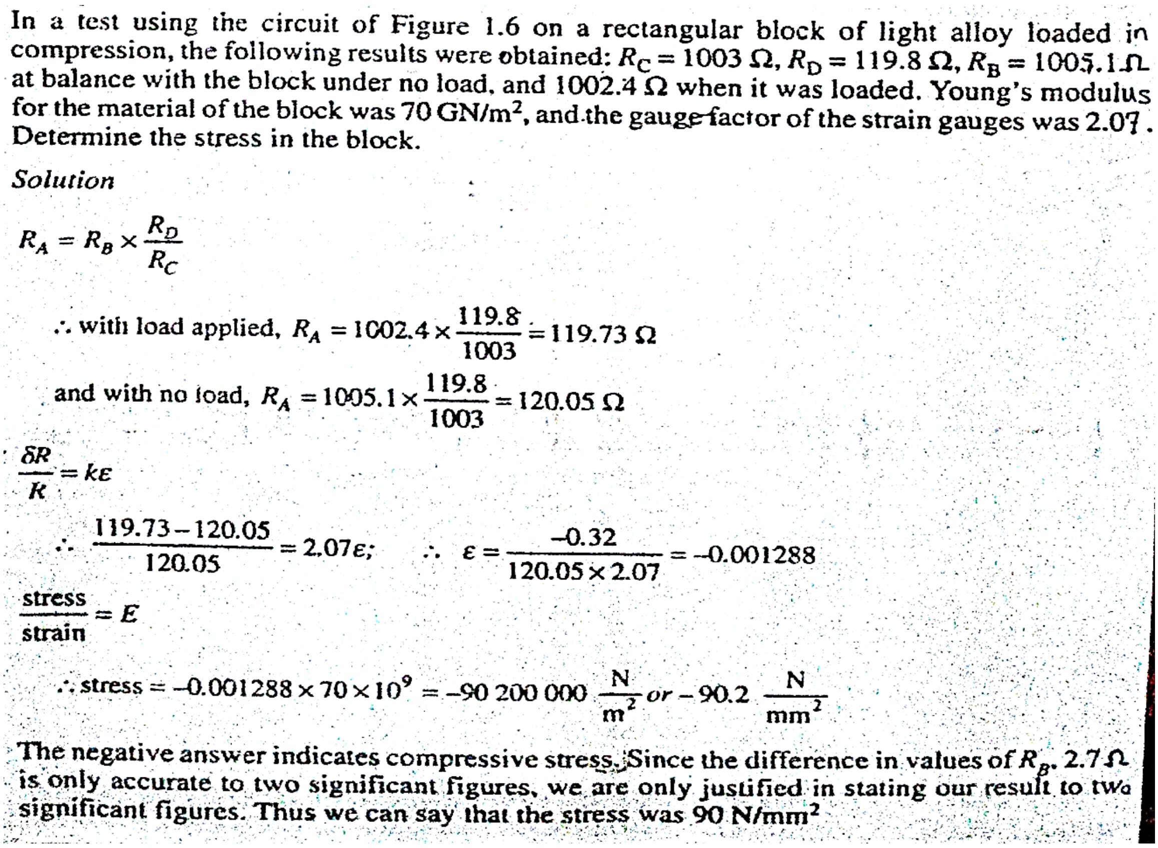
(This may also be written as 1300 microstrain. One microstrain is a strain of 0.000 001).

$$\therefore \frac{\delta R}{120.2} = 2.15 \times 1.3 \times 10^{-3}$$

$$\therefore \delta R = 120.2 \times 2.15 \times 1.3 \times 10^{-3} = 0.336\ \Omega$$

$$\therefore \text{percentage change} = \frac{0.336}{120.2} \times 100\% = 0.280\%$$

Example



In a test using the circuit of Figure 1.6 on a rectangular block of light alloy loaded in compression, the following results were obtained: $R_C = 1003\ \Omega$, $R_D = 119.8\ \Omega$, $R_B = 1005.1\ \Omega$ at balance with the block under no load, and 1002.4 Ω when it was loaded. Young's modulus for the material of the block was 70 GN/m², and the gauge factor of the strain gauges was 2.07. Determine the stress in the block.

Solution

$$R_A = R_B \times \frac{R_D}{R_C}$$

$$\therefore \text{with load applied, } R_A = 1002.4 \times \frac{119.8}{1003} = 119.73\ \Omega$$

$$\text{and with no load, } R_A = 1005.1 \times \frac{119.8}{1003} = 120.05\ \Omega$$

$$\frac{\delta R}{R} = k\varepsilon$$

$$\therefore \frac{119.73 - 120.05}{120.05} = 2.07\varepsilon; \quad \therefore \varepsilon = \frac{-0.32}{120.05 \times 2.07} = -0.001288$$

$$\frac{\text{stress}}{\text{strain}} = E$$

$$\therefore \text{stress} = -0.001288 \times 70 \times 10^9 = -90\,200\,000\ \frac{\text{N}}{\text{m}^2} \text{ or } -90.2\ \frac{\text{N}}{\text{mm}^2}$$

The negative answer indicates compressive stress. Since the difference in values of R_B, 2.7 Ω is only accurate to two significant figures, we are only justified in stating our result to two significant figures. Thus we can say that the stress was 90 N/mm²

Measurements and Instrumentation

Lecture 3

Temperature Transducers

Temperature

Is a function of the intensity of vibration of its atoms and molecules.

Temperature measuring unit:

- Celsius scale (centigrade scale) uses the Celsius degree as its unit of temperature.
  Celsius degree is one-hundredth of the temperature change between the freezing and boiling points of water at standard atmospheric pressure, the freezing point being 0^{o}C and the boiling point 100^{o}C.
- Kelvin is the same unit of temperature at as Celsius degree.
  Zero kelvin is absolute zero where it is the temperature at which atoms and molecules are absolutely vibrationless (at -273.15^{o}C). Thus the freezing and boiling points of water at standard atmospheric pressure are 273.15 K and 373.15 K respectively.

Methods of temperature measurement

There are four ways in which this can be done:

1. Expansion, temperature change can be converted into change of volume, as in a liquid-in-glass thermometer, or into change of shape, as in a bimetallic strip.
2. Thermoelectricity, a small voltage is generated where two different metals are joined together. The magnitude of the voltage generated depends on the temperature of the junction.
3. Electrical resistance, resistance of metals and of semiconductors varies with temperature.
4. Radiation, the heat energy radiated by a body increases in quantity and includes radiation at shorter and shorter wavelengths as temperature increases.

Thermocouples

If two different metals are joined together, either mechanically or by welding, a small continuous voltage is generated at the junction. This voltage is generated by what is known as the *Seebeck* or *thermoelectric* effect. It depends on the temperature of junction and on the metals used. Therefore, the junction is converting temperature into voltage.

- One of its applications is in gas-fire safety devices, see Figures 3.3, 3.4, 3.5.

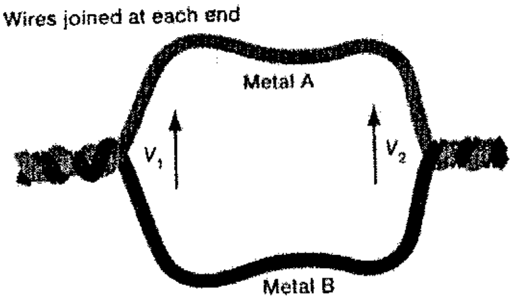


Figure 2.3 Basic thermocouple circuit

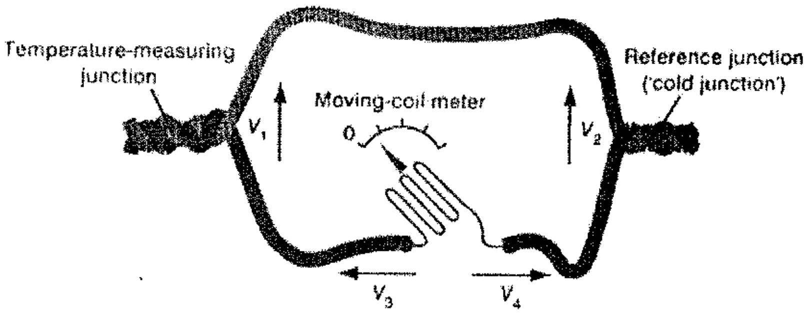


Figure 2.4 Insertion of a third metal into one of the thermocouple wires

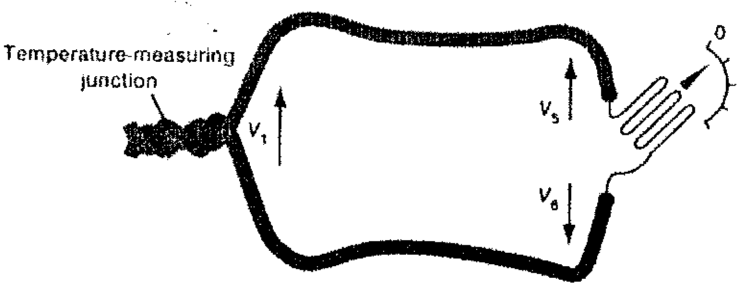


Figure 2.5 Replacing the reference junction by separate junctions with the third metal

Thermistors

This device is made from materials in which a large change of electrical resistance is produced by a small change of temperature. The usual type of thermistor has a negative temperature coefficient (n.t.c.) and a resistance-temperature relationship which is defined by:

$$R = Ae^{\frac{B}{T}}$$

Where: A and B are constants for the particular material, and T is absolute temperature of the thermistor in kelvins. Writing this equation for a particular pair of values of resistance and temperature, R_1 and T_1, we get:

$$R_1 = Ae^{\frac{B}{T_1}}$$

and dividing the general equation by the particular, gives:

$$R = R_1 e^{B(\frac{1}{T}-\frac{1}{T_1})}$$

From which the resistance R can be calculated at any absolute temperature T, if R_1, B and T_1 are known. B is temperature in kelvins, called the characteristic temperature of the thermistor, see Figure 2.6.

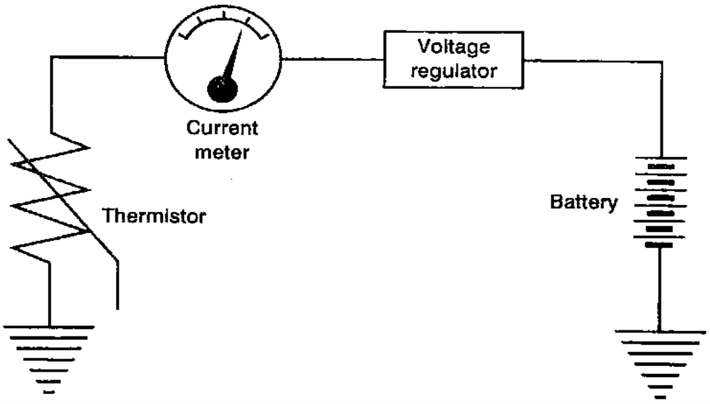


Figure 2.6 Circuit diagram for a motor car water temperature indicator system

- Application

One of its applications measures the temperature of the cooling water in a motor vehicle engine.

Positive Temperature Coefficient (PTC) Thermistors

In this type, increase of temperature increases its resistance, instead of decreasing it. It is usually used for over-temperature protection of industrial protection equipment, and for over-current protection of electrical circuits. Over-temperature protection type has a low resistance until the temperature reaches a specified reference temperature. Within a few degrees above temperature the resistance increases about a hundred-fold, switching off a circuit.

Over- current protection type is a switch-off fail-safe device, operated by the self-heating effect which occurs when too much current is passed through it.

The resistance of the thermistor increases rapidly if the current limit is exceeded, thus bringing the current down to a small fraction of the specified limit. A typical application is the protection of the primary winding of a mains transformer.

Calibration

There are two types of calibration:

1) Primary calibration, an instrument is calibrated by reference to values obtained from first principles under laboratory conditions.
2) Secondary calibration, an instrument is calibrated against values determined by a standard instrument which has itself received a primary calibration.

The bimetallic strip

This consists of two metals with unequal coefficients of linear expansion, bonded together to form a single piece of material. A change in temperature causes them to expand or contract unequally, and these results in a proportionate increase or decrease in the curvature of the material.

Simple dial-and-pointer type thermometers are made by forming the bimetallic strip into a spiral or a helix. This is fixed at one end to the body of the instrument and carries a pointer at the other end.

Temperature change causes the bimetallic strip to curl or uncurl so that the pointer indicates temperature on a circular dial.

Example 2.2

A probe, which consists of a thermocouple in a protective sheath, has a time constant of 4.0 seconds. It is to be used to measure the temperature of a molten metal. If its initial temperature is 20°C, and the temperature of the metal will not be greater than 650°C, determine the minimum time that the probe should remain in the molten metal, to obtain a reading within two degrees of the true value.

Solution

Step input = 650° − 20° = 630°C

$$\therefore e^{-\frac{t}{\tau}} = \frac{2}{630} = 0.00317$$

$$\therefore -\frac{t}{\tau} = -5.75$$

$$\therefore t = 5.75\tau = 5.75 \times 4.0 = 23.0 \text{ s}$$

Therefore the probe should remain in the molten metal for a minimum time of 23 seconds.

Example 2.3

A transducer with a time constant of 5.2 seconds measures the temperature of a liquid flowing at a constant rate into a chemical process. The recorded temperature is found to vary approximately sinusoidally between 216°C and 238°C, with a periodic time of 18 seconds. Determine the actual temperature variation and the time delay between corresponding points in the actual cycle and in the recorded cycle.

Solution

$$\text{Frequency} = \frac{1}{(\text{period})} = \frac{1}{18} = 0.0556 \text{ Hz}$$

$$\therefore \text{Angular frequency} = 2\pi \times 0.0556 = 0.349 \text{ rad/s}$$

$$\therefore \omega\tau = 0.349 \times 5.2 = 1.815$$

$$\text{Midpoint of temperature range} = \frac{238 + 216}{2} = 227°\text{C}$$

Amplitude of recorded temperature = 238 − 227 = 11°C

$$\frac{(\text{recorded amplitude})}{(\text{true amplitude})} = \frac{1}{\sqrt{1.815^2 + 1}} = \frac{1}{2.07}$$

True amplitude = 11 × 2.07 = 22.8°C

True temperature range is from 227 − 22.8 = 204°C to 227 + 22.8 = 250°C

Phase lag = arctan 1.815 = 61.1°

$$\text{Delay} = \frac{61.1}{360} \times 18 = 3.06 \text{ seconds}$$

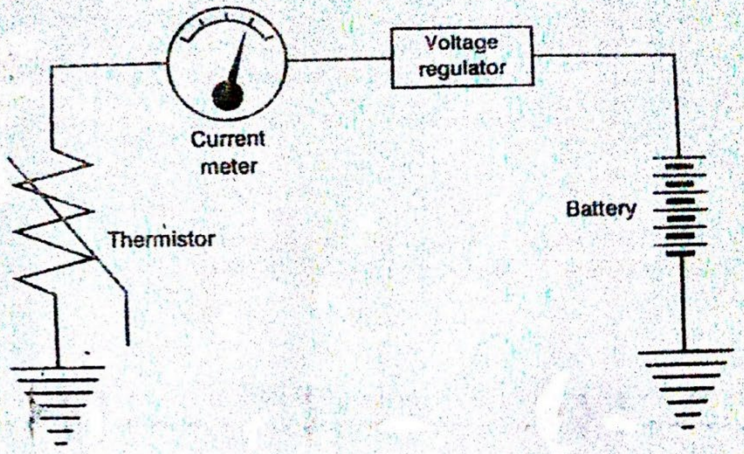


Figure 2.10 Circuit diagram for a motor car water temperature indicator system

Where more precise temperature measurement is necessary, we must allow for the fact that when we pass current through the thermistor it will heat up until its temperature is such that its output power (in the form of heat) is equal to the electrical power being dissipated in it.

Example 2.5

The dissipation factor of the thermistor in the previous example is given as 8.5 mW/°C. It is to be used for temperature measurement over the range 0°C to 100°C. If its temperature is not to differ by more than 1.5°C from the temperature of its surroundings, calculate:

a) the maximum constant voltage which can be applied to it.
b) the resulting current range corresponding to the temperature range.

Solution

a) Maximum allowable power $= 1.5 \times 8.5 = 12.75\,mW = 0.01275\,W$.

Electrical power $P = \frac{V^2}{R} \quad \therefore V = \sqrt{P \times R}$

Therefore for a given value of power transfer, the maximum constant voltage is limited by the least possible resistance the thermistor can have within the given temperature range, that is, 103 Ω (from the table on p.27)

Then maximum constant voltage $= \sqrt{0.01275 \times 103} = 1.15\,V$

b) At this voltage, by Ohm's law, the current through the thermistor will vary from:

$$\frac{1.15}{103} = 0.0112\,A = 11.2\,mA \text{ at } 100°C$$

$$\text{to } \frac{1.15}{5090} = 0.000226\,A = 0.226\,mA \text{ at } 0°C$$

If the thermistor is to be enclosed in a protective coating or sheath however, its dissipation factor, and therefore its voltage and current limits, will be considerably reduced. It would then be better to put up with the unknown temperature error and calibrate the system by applying known temperatures to it and plotting the corresponding current values.

Measurements and Instrumentation
Lecture 4
Displacement Transducers

Mechanical devices

These devices usually consist of combination of rack and pinion, gear train, cable and drum and lever. Their purpose is to amplify small displacement.

- Advantage:

It is a positive in action – that is, the amplification factor, is set fixed and cannot vary.

- Disadvantages:

1) Input displacement must be applied with sufficient force to overcome friction in the mechanism.

2) Inertia of the components of the mechanism is magnified by the gain of the mechanism, so very large input forces are needed if the device is accurately to follow rapidly varying displacements.

Purely mechanical devices are not remote reading.

Capacitive transducers

This is the differential pressure transducer. The two pressures, P_1 and P_2, are applied each to an isolating diaphragm at either end of the transducer body.

The diaphragms deflect and transfer the pressure through silicone oil, which also acts as the dielectric, to the opposite faces of the sensing diaphragm.

Deflection of the sensing diaphragm towards one of the fixed plates reduces the thickness of dielectric on that side, increasing its capacitance, thus reducing its reactance and vice-versus, see Figure 3.7.

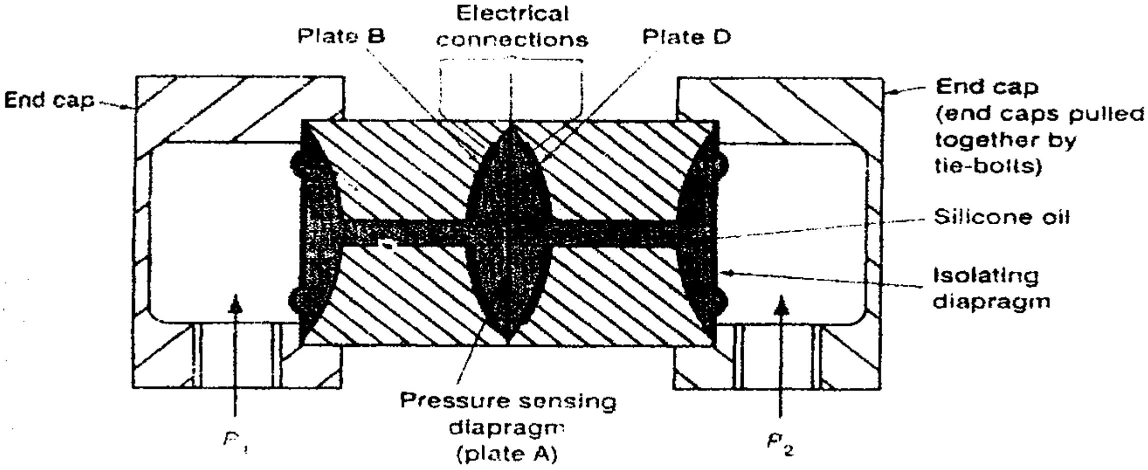


Figure 3.7 Section through capacitive pressure transducer

- Application:

  It can be used to measure the contents of fuel tanks in aircraft.

- The main advantage, the total capacitance is made up of a number of such pairs of tubes at suitable places in the tank, the reading is largely unaffected by the attitude of the aircraft in pitch or roll.

Binary coded disc

This component can be used to measure angular or linear displacement. It is a disc that divided to sectors with different colors (black and white) which represent the binary logic (‘0’,’1’), see Figure 3.8.

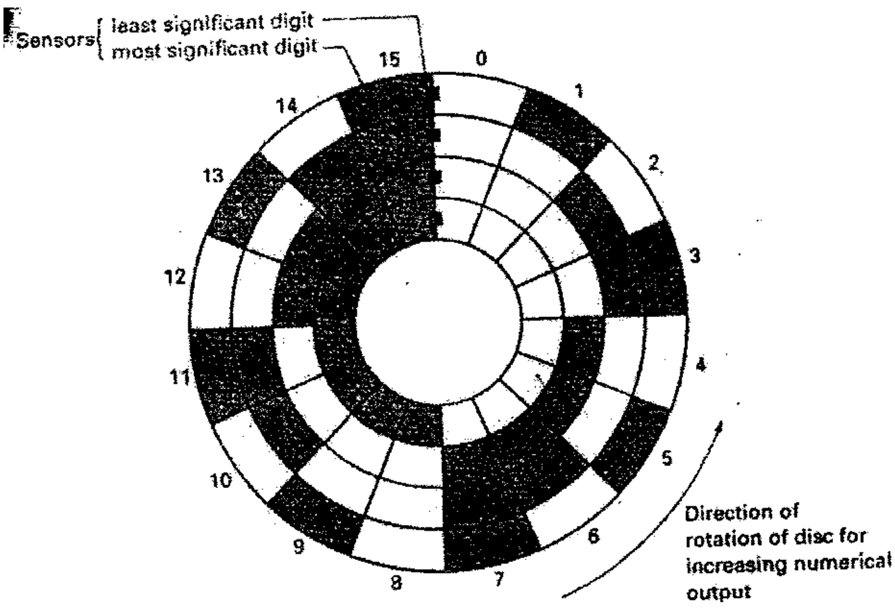


Figure 3.8 Disc encoded with the natural binary sequence from 0 to 15

Overcoming ambiguity

Binary coded disc problem is that when more than one binary digit changes the sensors do not detect the change simultaneously.

There are two methods to manipulate this problem:

1) To add an anti-ambiguity track and an extra sensor, see Figure 3.9.

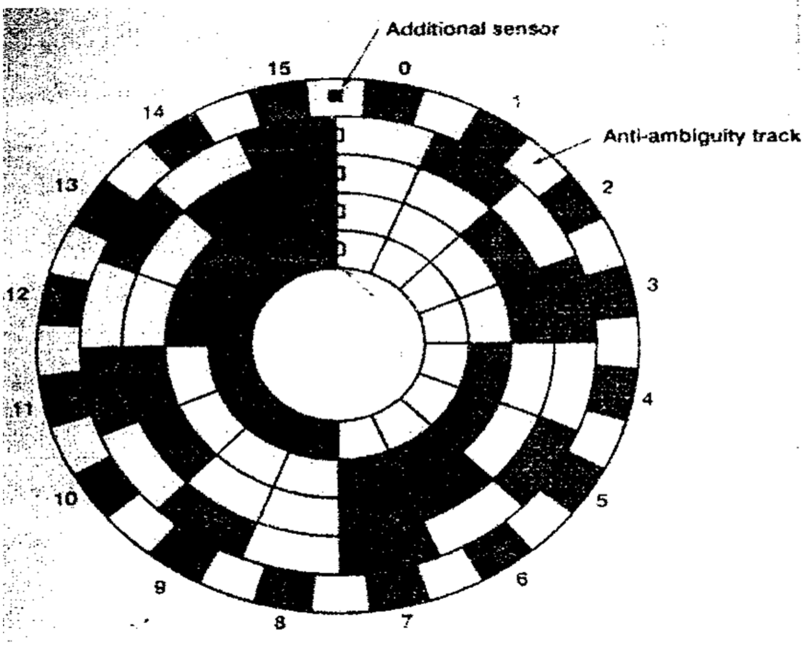


Figure 3.9 The disc with an – ambiguity track added

2) Using Gray cyclic coded pattern on the disc instead of natural binary pattern, see Figure 3.10.

The benefit of Gray code that only one digit at a time changes.

To measure an angle of rotation to the nearest 1/16 of a revolution, to the nearest 22.5°. Each disc has four tracks, because $16 = 2^4$. To measure rotation with an error not greater than one degree, the disc need to have nine tracks (because $2^8<360<2^9$).

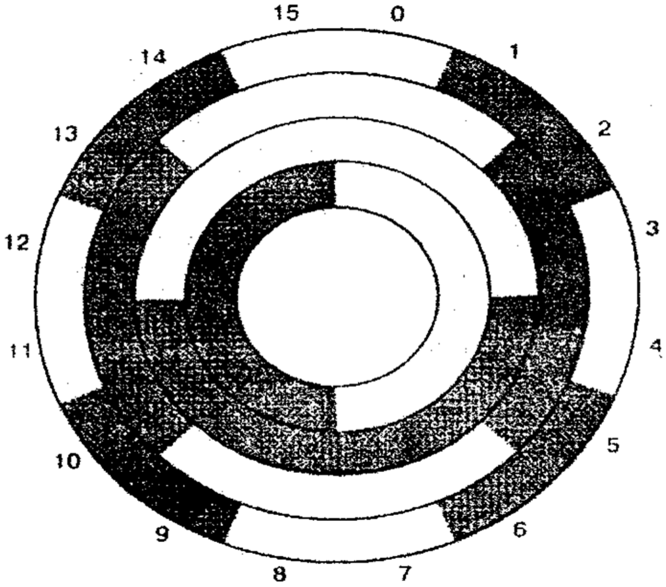
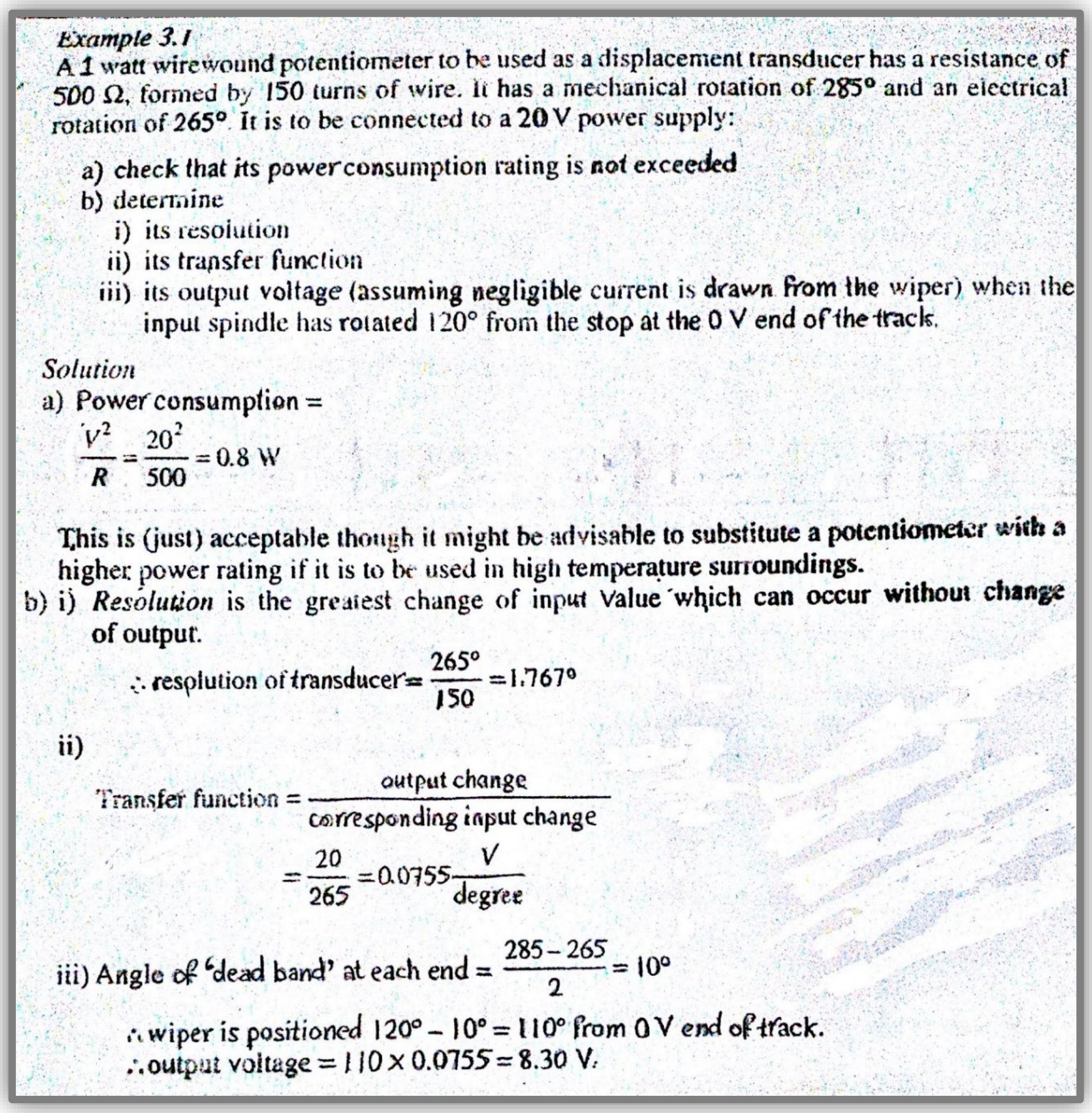


Figure 3.10 Disc coded with gray code



Example 3.1

A 1 watt wirewound potentiometer to be used as a displacement transducer has a resistance of 500 Ω, formed by 150 turns of wire. It has a mechanical rotation of 285° and an electrical rotation of 265°. It is to be connected to a 20 V power supply:

a) check that its power consumption rating is not exceeded

b) determine

  i) its resolution

  ii) its transfer function

  iii) its output voltage (assuming negligible current is drawn from the wiper) when the input spindle has rotated 120° from the stop at the 0 V end of the track.

Solution

a) Power consumption =

$$\frac{V^2}{R} = \frac{20^2}{500} = 0.8\ \text{W}$$

This is (just) acceptable though it might be advisable to substitute a potentiometer with a higher power rating if it is to be used in high temperature surroundings.

b) i) ***Resolution*** **is the greatest change of input value which can occur without change of output.**

$$\therefore \text{resolution of transducer} = \frac{265^\circ}{150} = 1.767^\circ$$

ii)

$$\text{Transfer function} = \frac{\text{output change}}{\text{corresponding input change}}$$

$$= \frac{20}{265} = 0.0755 \frac{V}{\text{degree}}$$

iii) Angle of 'dead band' at each end $= \dfrac{285 - 265}{2} = 10^\circ$

$\therefore$ wiper is positioned $120^\circ - 10^\circ = 110^\circ$ from 0 V end of track.

$\therefore$ output voltage $= 110 \times 0.0755 = 8.30$ V.

Example 3.2

A displacement measurement system is to be made up of a 1 kΩ wirewound potentiometer with its output connected to a moving coil voltmeter. The voltmeter has a range of 0 to 15 V and a resistance stated as 1 kΩ/V. The power supply is 15 V and the potentiometer is rated at 1.5 W.

a) Check that the power rating of the potentiometer will not be exceeded.

b) Calculate the percentage error in the output of the potentiometer at mid-range due to potentiometer loading.

c) Suggest how the accuracy of the system could be improved.

Solution

a) Power dissipated in the potentiometer =

$$\frac{V^2}{R} = \frac{15^2}{1000} = 0.225\ \text{W}$$

which is well within the rating of 1.5 W.

b) We could use the final equation above, but it is quite easy to work from first principles: 1 kW/V on a range of 0 to 15 V gives a voltmeter resistance of 1000 × 15 = 15 000 Ω. Referring to Figure 3.2, at mid-range of the potentiometer the wiper divides the potentiometer resistance into two equal portions of 500 Ω each. The voltmeter resistance is in parallel with the lower portion. The effective resistance of 500 Ω and 15 000 Ω in parallel is:

$$\frac{500 \times 15\,000}{500 + 15\,000} = 483.9\ \Omega$$

Therefore the output voltage of the potentiometer at mid-range is:

$$\frac{483.9}{500 + 483.9} \times 15 = 7.377\ V$$

At mid-range, the output voltage of the potentiometer would be 7.5 V if the system were linear. Therefore the percentage error is:

$$\frac{7.5 - 7.377}{7.5} \times 100 = 1.64\%$$

c) The following improvements could be made:

1 Instead of a wirewound potentiometer, use one with a conductive plastic track. This would have infinite resolution, and the linearity of its resistance element would be better.

2 Instead of a moving coil voltmeter use a digital voltmeter. This would have an input resistance of (typically) 20 MΩ, giving negligible non-linearity when used as the load on the potentiometer output, and its accuracy and resolution would be much better than that of an analogue meter.

Measurements and Instrumentation

Lecture 5

Force, Torque and Pressure Transducers

Hydraulic load cell

In this transducer, the force is applied to a diaphragm (moving part) that covers a capsule filled by oil. The resulting pressure measured by a pressure gauge graduated in units of force, see Figure 4.1.

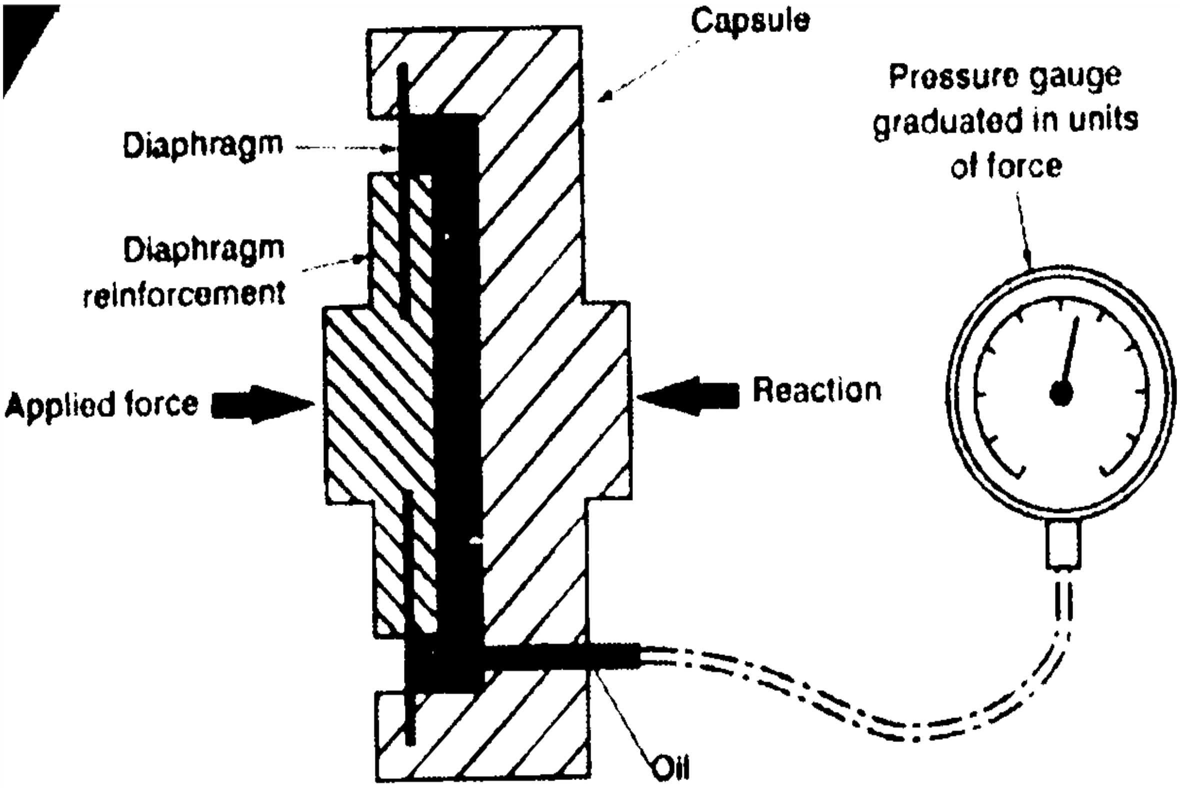


Figure 4.1 Section through a hydraulic load cell

Piezoelectricity

In this transducer which used quartz or other types of piezoelectricity material, this material acquires electrostatic charge proportional to the load when it is loaded in compression and a corresponding voltage appears between the electrodes. It can thus be used as a force transducer, see Figure

4.2.

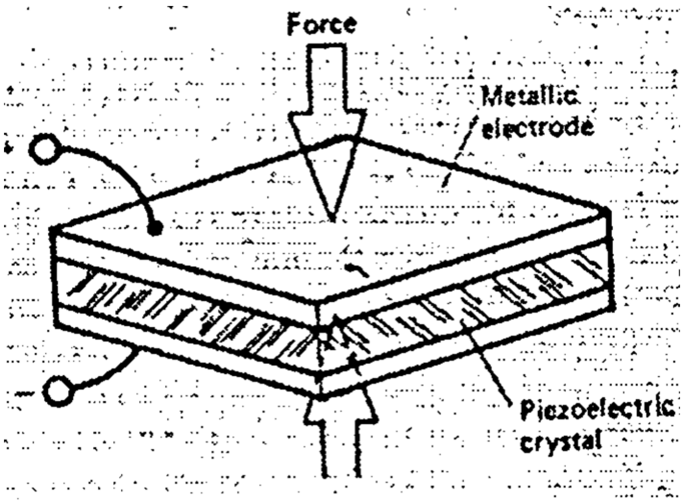


Figure 4.2 The piezoelectric effect

Torque measurement

Measurement of torque in a rotating shaft

1) Strain gauge: The difficulty in this method, how to make electrical connection to it. This can be done by following method:
   a) Slip ring: The main problem is the friction. Therefore, the method used to temporary measurements.
   b) Rotary transformer, see Figure 4.7.

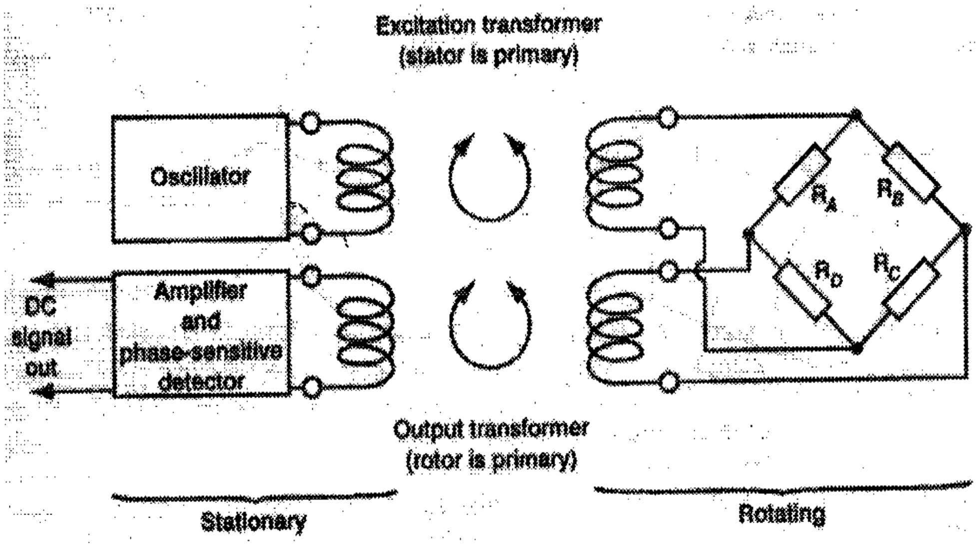


Figure 4.7 Rotary transformer circuit for supplying power to strain gauges

on a rotating shaft and receiving their output

c) Radio telemetry

These two methods (b, c) depend on excitation transformer and radio signal to transfer signal from rotating shift. Since there is no connection between surface it can be used to permanent installations.

2) Torsional deflection

a) Use the principle that one end of a shaft twists relative to the other in proportion to the torque applied., see Figure 4.8.

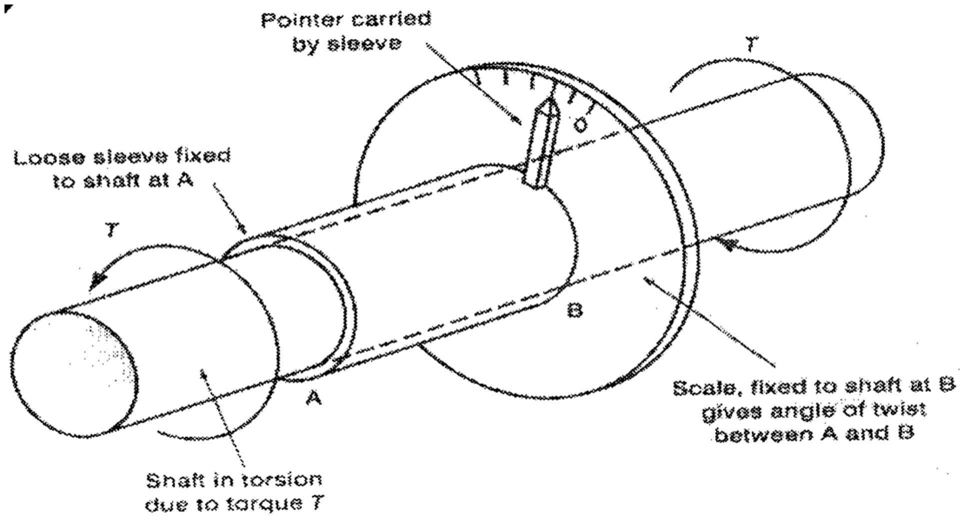


Figure 4.8 Torque measurement from angle twist of shaft

b) Two radially slotted discs are attached to the shaft, a small distance apart. A non-rotating light source and photodetector are used to measure the illumination received through the slots in the discs. The slots are exactly aligned when the shaft is under no torque. As torque is applied, the shaft twists, causing one disc to rotate slightly relative to the other.

The light received by the photodector is reduced in the same proportion as the torque applied. Provide that a photodetector with a short rise time is used, its output when the slots pass through the beam of light is the same as when the shaft is stationary, see Figure 4.9.

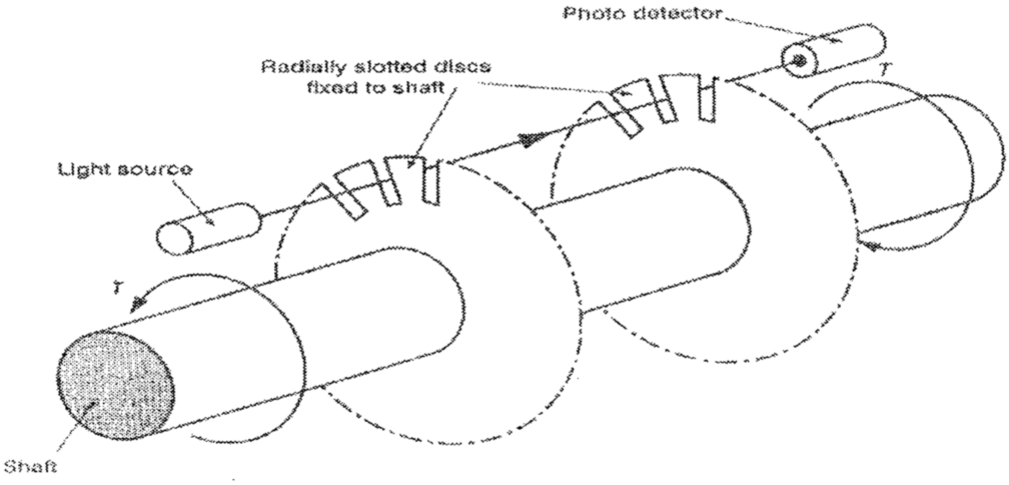


Figure 4.9 Torque measurement by measuring the illustration through the slots of a pair of radially slotted disc

Pressure measurement

In this type, internal pressure causes the flattened cross-section to open out to a more circular form, and this causes tube itself to tend to straighten out. Change the shape of this type of measuring device gives different advantage:

a) Using a diaphragm for low pressure.

b) Twisted to be more resist to shock and vibration.

c) Lengthened and formed into a spiral or helix to increase sensitivity.

See Figure 4.11.

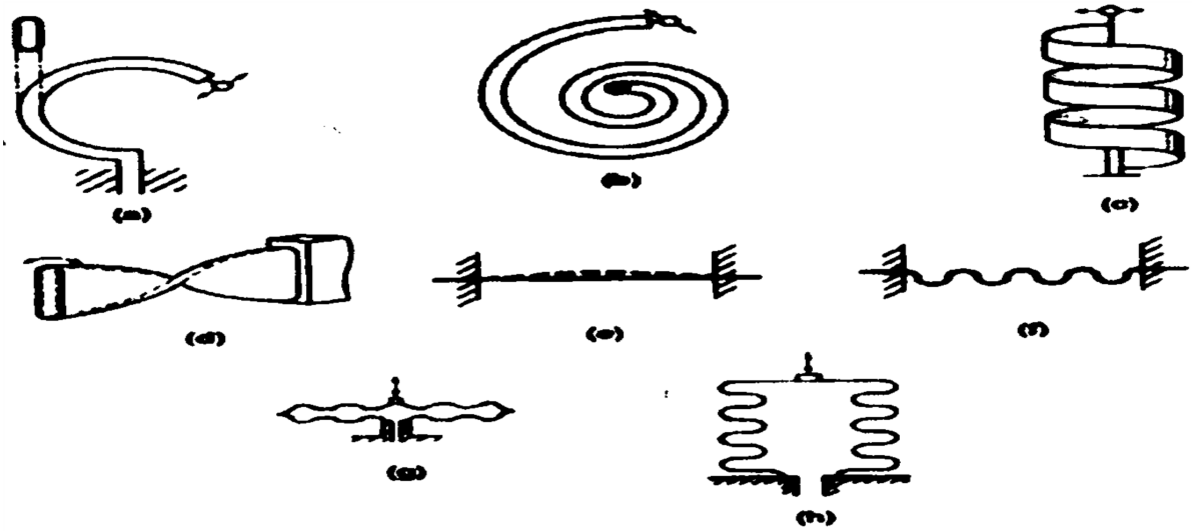


Figure 4.11 Transducer which give displacement proportional to pressure:

(a) C – type, (b) Spiral, (c) helical, (d) twisted. Diaphragms (e) flat, (f) corrugated capsule, (g) Bellows (h).

U-tube manometers

These work on the principle that at the lower of the two levels in the U-tube, the pressures in the two limbs are equal, so the difference between the pressures acting on the two surfaces is given by: $p_1 - p_2 = \rho g h$

Where:

ρ is the density of the liquid

g is gravitational acceleration 9.8 m/s^2

h is the height of liquid above the lower level.

Example 4.2

a) A piezoelectric force transducer has a sensitivity of 28 pC/N. It is connected to a charge amplifier which has a feedback capacitor of 22 nF. Calculate the output voltage of the charge amplifier at the instant when a step input of 5 kN is applied to the transducer.

b) The system of piezo transducer and charge amplifier has a time constant of 90 seconds. How long will it take to lose the first 5% of the output step?

Solution

a) The load of 5 kN charges the transducer with a charge of:

$$q = 28 \times 5000 = 140\,000 \text{ pC} = 140 \text{ nC}$$

$$V_o = -\frac{q}{C_f} = \frac{140 \times 10^{-9}}{22 \times 10^{-9}} = 6.36 \text{ V}$$

(The negative sign in the original equation merely means that the output voltage of the charge amplifier will be of opposite polarity to its input voltage.)

b) Figure 4.5 show the exponential decay of the output voltage of the charge amplifier after the step.

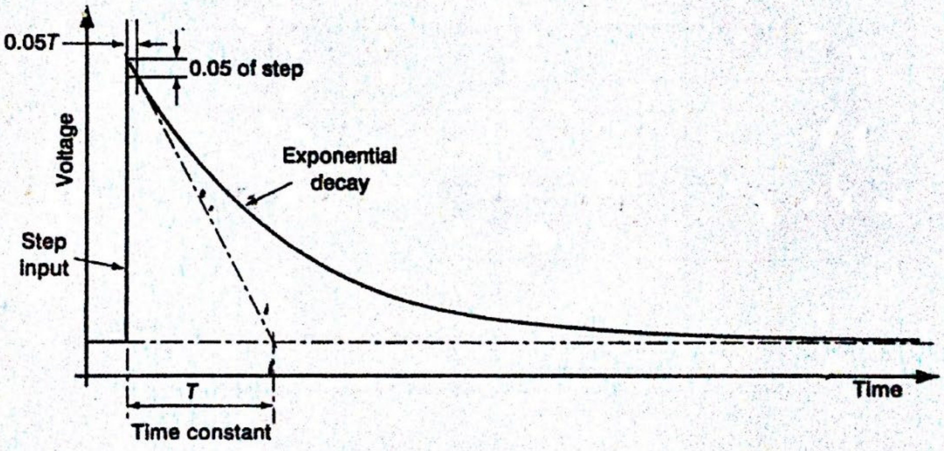


Figure 4.5 Exponential decay of the output of a charge amplifier

By the approximation on p.20, 0.05 of the step will have been lost in a time of 0.05τ, that is, in 0.05 × 90 = 4.5 seconds. Or, more accurately, using

$$v = Ve^{t/\tau}$$

$$0.95V = Ve^{t/\tau}$$

$$\therefore \ln(0.95) = -\frac{t}{90}$$

$$\therefore t = -90 \times \ln(0.95) = 4.61 \text{ seconds}$$

Measurements and Instrumentation

Lecture 6

Acceleration and Vibration Transducers

Seismic pickups

It is used to measure the motion of the surfaces to which they are fixed. They are sensitive to motion along one axis only, so if the motion is three dimensional, three seismic pickups are needed to determine the components of the motion along three mutually perpendicular axes.

The essential component is the seismic mass. This is a body of metal, suspended from a resilient support. This is a support whose deflection is proportional to the force applied to it.

The inertia of the seismic mass causes it to lag behind the motion of the casing when the casing is accelerated, causing a deflection in the support. This deflection forms the input to a transducer, which produces a proportional output signal. The transducer is represented by a potentiometer, but any suitable type of transducer may be used (unbounded strain gauge bridge), see Figure 6.1.

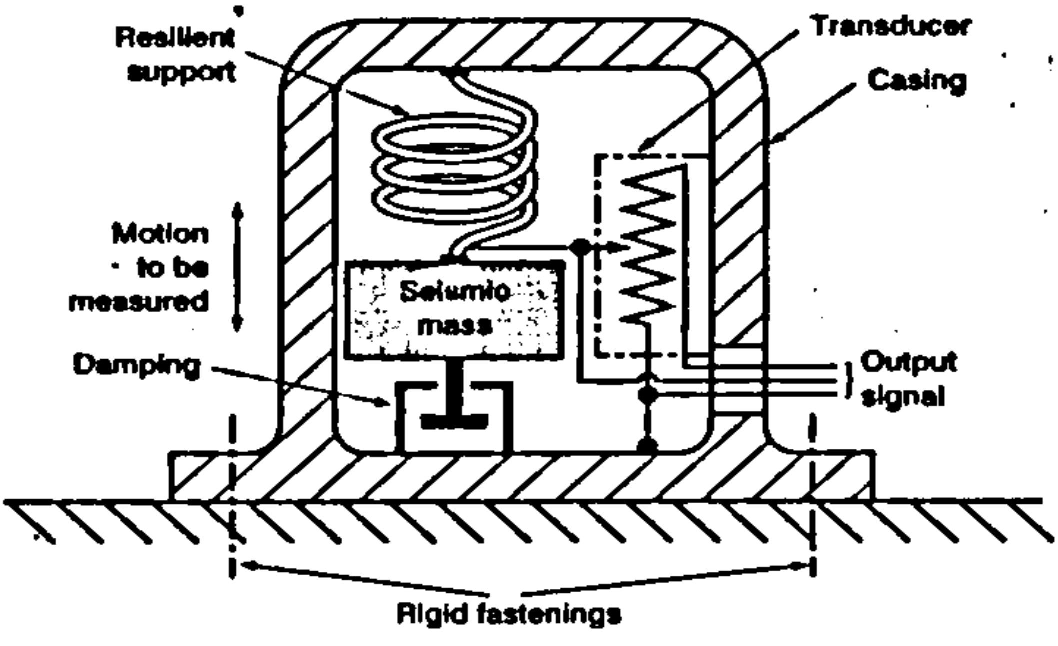


Figure 6.1 The essential features of a seismic pickup.

Displacement pickups

This type of pickup is used to measure the displacement of a vibrating body when there is no fixed reference point available, for example in determining the movement of the chassis of a vehicle. Therefore, the seismic mass to behave (as far as possible) as though it was fixed in space.

Acceleration pickups (accelerometer)

For designing the pickup system to have a low value of ω_n, it can be used as displacement or a velocity pickup for angular frequencies well above ω_n.

To design it as an acceleration pickup it must go to the opposite extreme.

To give it these accelerations, corresponding forces must be applied by the spring because:

Force = mass × acceleration. Therefore, the spring used as a transducer, to measure the force applied to the mass, its acceleration, and hence the acceleration of the casing.

Piezoelectric accelerometers

Tension cannot be applied to a crystal without using some kind of adhesive to make the tensile connection, and such a connection would be unreliable, so the crystal is kept permanently compressed by the seismic mass.

Thus the effect of acceleration in alternate directions is an alternating increase and decrease in the compressive fore on the crystal. The compressive preload is applied by screwing the seismic mass down on to the crystal to a given torque. The construction gives an accelerometer which is rugged, but because the casing is part of the ‘spring’ in the spring mass system, it may be subject to spurious input. These include temperature change acoustic noise, base bending, cross-axis motion, and magnetic fields.

Decreasing the seismic mass has the advantage of increasing the undamped natural frequency, and hence of increasing the frequency range of the accelerometer, but has the disadvantage of seriously reducing sensitivity, because the electrical output comes from the work done by the seismic mass.

However, a reduced sensitivity may be acceptable if the accelerometer is only being used to find the natural frequency of vibration of a structure by finding the frequency of excitation at which the output of the accelerometer is a maximum.

Measurements and Instrumentation

Lecture 7

Flow Measurement

The venturi meter

At the throat of the venturi meter, the pressure is reduced because the velocity of the liquid is greater in the smaller cross-section, this pressure drop is regained as the cross-section enlarges again to the original pipe diameter. Pressure tappings at the entry to the venturi and at the throat are led to a differential pressure transducer, from which a signal proportional to the pressure difference may be obtained, see Figure 7.1.

The pressure difference in a venturi meter is governed by Bernoulli's law; that in any continuous body of liquid the sum of potential energy, pressure energy and kinetic energy is constant at all points. In a horizontal pipe potential energy is constant and thus cancels out from any equation.

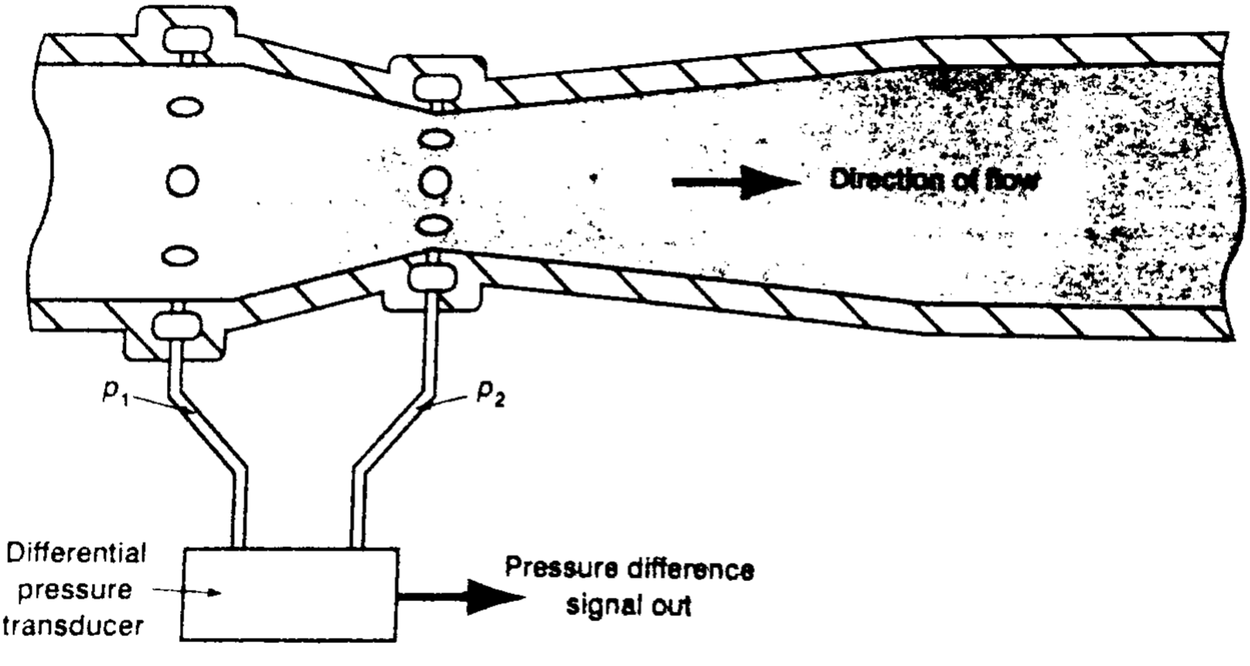


Figure 7.1 A venturi meter

The sum of pressure and kinetic energy at the upstream tapping and at the throat for unit mass (1 kg) of liquid, the following equation is obtained:

$$\frac{p_1}{\rho}+\frac{v_1^2}{2}=\frac{p_2}{\rho}+\frac{v_2^2}{2}$$

Where:

p is pressure (N/m^2), v is velocity (m/s), ρ is the density of the liquid (kg/m^3)

The volume flow rate:

$$\dot{Q}=a_1v_1=a_2v_2\,\frac{m^3}{s}$$

Where:

a = is the cross – sectional area of flow (m^2)

Furthermore:

$$\dot{Q}=\frac{a_1a_2}{\sqrt{(a_1^2-a_2^2)}}\sqrt{\frac{2}{\rho}(p_1-p_2)}\;\;{m^3}/{s}$$

This can be simplified:

$$\dot{Q}=\;a_2\sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$$

Where:

$$\beta\ is\ ratio{:}\frac{throat\ diameter}{pipe\ diameter}$$

To obtain the actual flow rate it is multiplied by a coefficient of discharge, C_d , which is found in range 0.97 to 0.99.

Reynolds number

This number gives indication whether the flow in the pipe is laminar or turbulent.

If Reynolds number (Re) is less than 2000, the flow in a pipe is laminar vice versus

it is turbulent. Reynolds number may be calculated as:

$$Re=\frac{vd\rho}{\eta}=\frac{vd}{v}$$

Where:

v is velocity of the liquid (m/s)

d is the inside diameter of the pipe (m)

ρ is the density of the liquid (kg/m^3)

ɳ (eta) is the dynamic viscosity of the liquid (Ns/m)

v (nu) is the kinematic viscosity of the liquid (m^2/s)

The Reynolds number of the flow in the throat of a venture is always greater than that in the pipe because, for a given flow rate of a given liquid, Reynolds number is inversely proportional to diameter, because:

$$Re \propto v \times d \propto \frac{1}{a} \times d \propto \frac{1}{d^2} \times d \propto \frac{1}{d}$$

So we need only consider the Reynolds number of the flow in the pipe itself. The coefficient of discharge of a venturi meter is only constant for Reynolds numbers

greater than 2×10^5

The hot-wire anemometer

It consists of a tine tungsten wire stretched between the tips of streamlined support. It measures velocity from the effect, on the wire's electrical resistance, of the cooling caused by the flow of the fluid past the wire.

It is mainly used to measure the flow velocities of gases but it can also be applied to the measurement of liquid velocities.

Because of small thermal capacity it is particularly useful for the measurement of rapid fluctuations in velocity.

The heat transfer from the wire to the fluid is determined by the following equation:

$$\textit{Heat transfer rate (watts)} = A + B\sqrt{v}$$

v is the velocity of flow while A and B are constants.

Equating heat power output to electric al power input gives:

$I^2R = A + B\sqrt{v}$

Where

I is the current through the wire and R is its resistance.

The hot wire anemometer is used with a bridge circuit. The best method is to keep the resistance of the hot wire constant, by adjusting the bridge power supply voltage to keep the bridge balanced.

There are two practical difficulties which could occur with a hot-wire anemometer:

1) It may vibrate in high flow velocities, causing it to break.
2) It may become coated with dirt, which will alter the calibration, or it may even be broken by impact with large dirt particles.

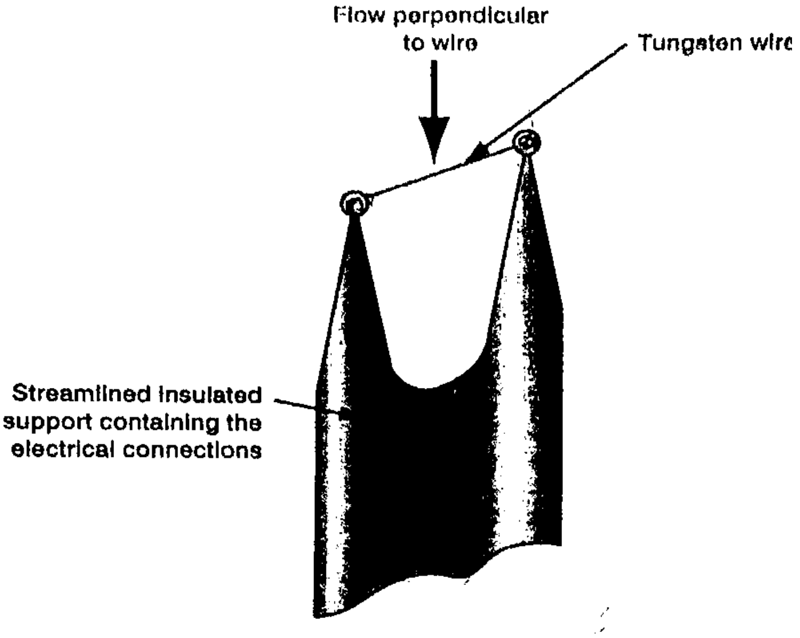


Figure 7.7 Hot-wire anemometer

The calibration of flowmeters

In industry application, the fluid flow is always avoided because the calibration of a flowmeter can be affected by variations in the density and viscosity of the fluid as its temperature and pressure change and by flow disturbances due to elbows, tees or valves immediately upstream or downstream of the flowmeter. Accurate calibration is important because the cost control of plant operation depends largely on flow measurement.

The primary calibration of a flowmeter for a liquid is done by setting a steady rate of flow and measuring the time taken to collect a given volume of the outflow in a tank.

For a gas, it is collected over water in a gasometer, a counterbalanced inverted cylindrical container which rises as the gas is accumulated. The temperature and pressure of the gas must be measured so that the calibration can be corrected for variation in gas density, see Fig. 7.10.

Secondary calibration of a flowmeter may be carried out by connecting it in series with another flowmeter which has already been calibrated, and using the flow rates measured by the calibrated flowmeter to calibrate the uncalibrated one.

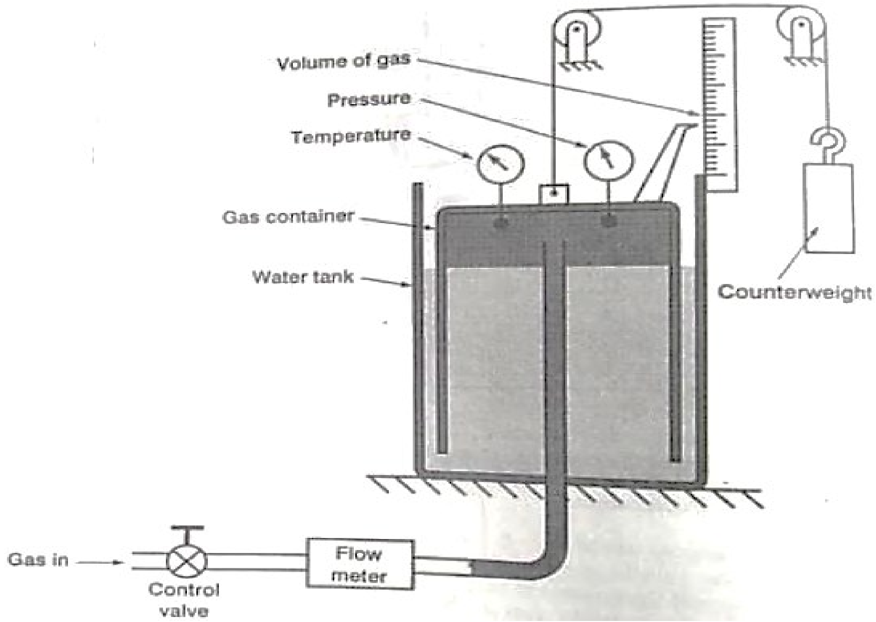


Figure 7.10 Calibration of a gas flowmeter

Example 7.1

A pipe of 100 mm internal diameter carries water at 20°C, which has a density of 998 kg/m^3 and a dynamic viscosity of 1.002 cP Calculate the velocity:

a) below which the flow will be completely laminar

b) above which the flow will be completely turbulent.

Solution

$$\text{a)}\quad Re = \frac{vd\rho}{\eta} \therefore 2000 = \frac{v \times 0.1 \times 998}{1.002 \times 10^{-3}} \quad \frac{\cancel{m}}{\cancel{s}} \times \cancel{m} \times \frac{kg}{\cancel{m}^3} \times \frac{\cancel{m}\,\cancel{s}}{kg}$$

$$\therefore v = \frac{2000 \times 1.002 \times 10^{-3}}{0.1 \times 998} = 0.0201 \text{ m/s}$$

$$\text{b)}\quad v = \frac{0.0201 \times 2 \times 10^5}{2000} = 2.01 \text{ m/s}$$

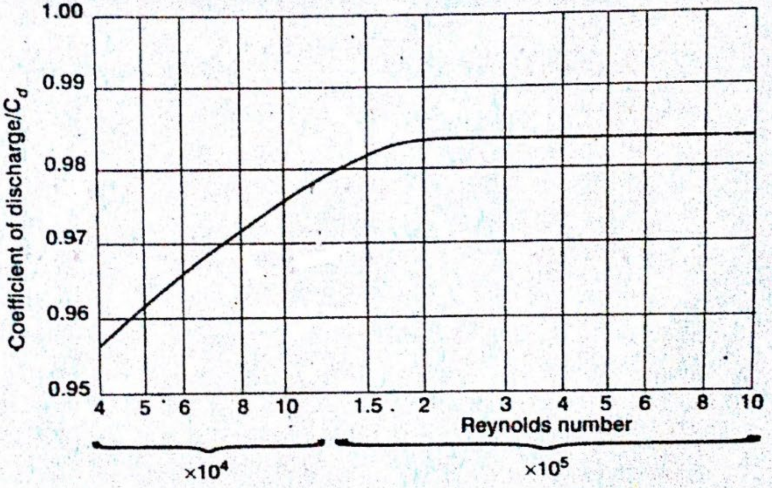


Figure 7.2 Coefficient of discharge of a typical venturi meter in a 100 mm diameter pipe, plotted on a logarithmic scale of Reynolds number

The above example shows that in a pipe of reasonable size, water at room temperature would have to be moving quite slowly for the flow to be laminar; usually flow velocities exceed the 2 m/s obtained in the answer to b) above.

Example 7.2

The throat and full bore diameters of a horizontal venturi meter are 30 mm and 60 mm respectively.

a) Calculate the coefficient of discharge of the venturi meter if the pressure at the full bore section is 47.7 kN/m^2 above that at the throat when water is being pumped through it at the rate of 0.420 m^3/min. Take the density of water as 1000 kg/m^3.

b) If a liquid with a relative density of 0.75 and a dynamic viscosity of 0.00157 Ns/m^2 is pumped through the same venturi meter, calculate the flow rate which would give the same Reynolds number, and hence the same coefficient of discharge. Take the dynamic viscosity of water as 0.001 Ns/m^2.

Solution

a) $$\dot{Q} = C_d a_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$$

$$a_2 = \frac{\pi}{4} \times 0.030^2 = 0.000707\ \text{m}^2; \quad \beta = \frac{30}{60} = 0.5$$

$$\therefore \frac{0.420}{60} = C_d \times 0.000707 \times \sqrt{\frac{2 \times 47700}{1000(1-0.5^4)}} \quad \therefore 0.007 = 0.00713 C_d$$

$$\therefore C_d = 0.982$$

b) $$a_1 = \frac{a_2}{\beta_2} = \frac{0.000707}{0.25} = 0.00283\ \text{m}^2$$

$$v_1 = \frac{\dot{Q}}{a_1} = \frac{0.007}{0.00283} = 2.47\ \text{m/s}$$

∴ in part a)

$$\text{Re} = \frac{vd\rho}{\eta} = \frac{2.47 \times 0.060 \times 1000}{0.001} = 148200$$

For the other liquid:

$$148200 = \frac{v \times 0.060 \times 0.75 \times 1000}{0.00157}$$

$$\therefore v = \frac{148200 \times 0.00157}{0.06 \times 750} = 5.17\ \text{m/s}$$

Flow rate:

$$a_1 v_1 = 0.00283 \times 5.17 = 0.01463\ \text{m}^3/\text{s}$$

or

$$\mathbf{0.01463 \times 60 = 0.878\ m^3/min.}$$

Measurements and Instrumentation

Lecture 8

Electronics for Instrumentation

Amplifier

An electronic circuit which increases the voltage of a signal.

- Features

1) Has i/p impedance (Z_{in}).
2) Has o/p impedance (Z_{out}).
3) DC power supply.

, see figure 8.2.

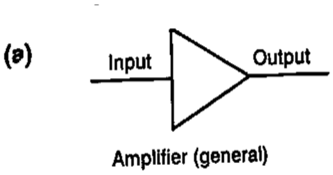


Figure 8.2 The amplifier circuit diagram circuit

Q: In small signal form systems which type of amplifier used?

Ans.: Buffers: main features:

1) Gain is approximately unity.
2) The amplifier is non-phase inverting.
3) The i/p impedance high.
4) The o/p impedance low.

Q: In large signal form system which type of amplifier used?

Ans.: power amplifier.

Q: Compare between AC amplifier and DC amplifier?

Ans.:

AC	DC
1) Coupling of internal stages of amplifier being through capacitors or transformers.	**1)** Direct connection between internal stages of amplifier.

2) Low frequency limit to bandwidth.	**2)** Frequency range extend down to zero.

Operational Amplifier (Op-Amp)

An operational amplifier (op-amp) is a high-gain amplifier with differential inputs. Its circuit diagram symbol is shown in Figure 8.3. It has two input connections; one, marked ‘-’ on the symbol, is the inverting input, the other, marked ‘+’, is the non-inverting input.

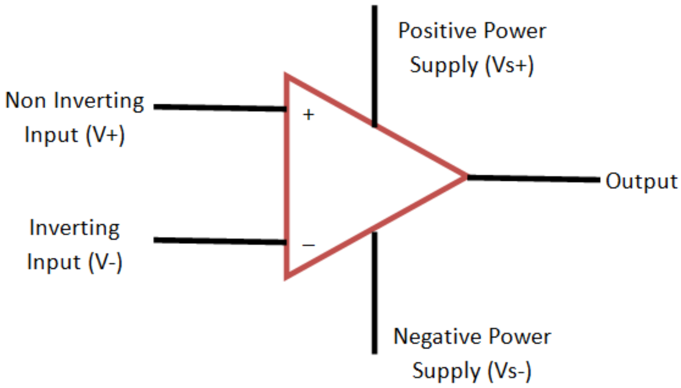


Figure 8.3 Symbol for an essential amplifier

- ➢ Types:

1) Inverting, see Figure 8.4.

$$gain = -\frac{R_f}{R_1}$$

$$\frac{V_{out}}{R_f} = -\frac{V_{in}}{R_1}$$

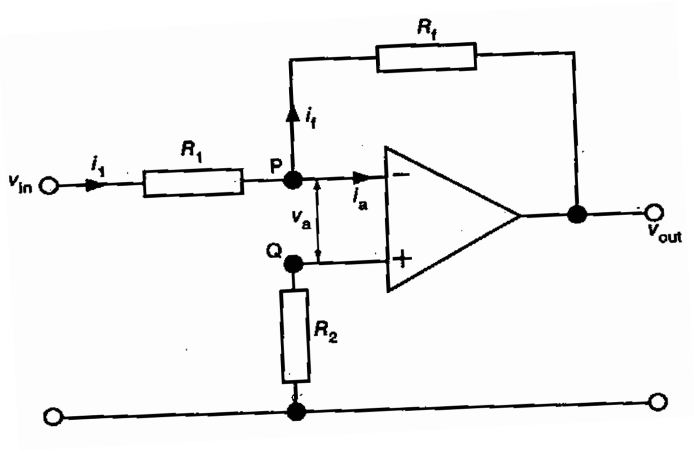


Figure 8.4 An inverting amplifier

2) Non-inverting, see Figure 8.5

$$gain = \frac{R_1+R_f}{R_1}$$

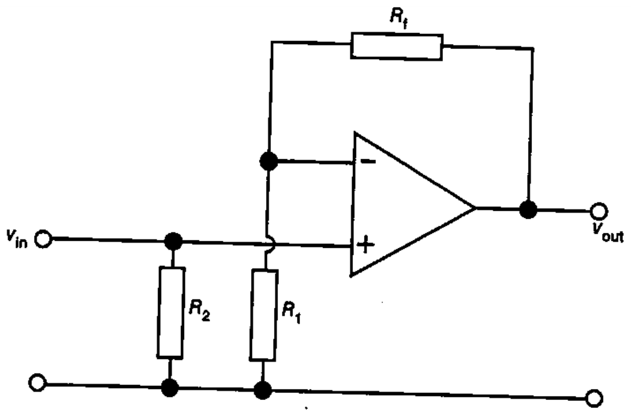


Figure 8.5 A non – inverting amplifier

Q: In inverting and non-inverting amplifiers R_2 is normally equal to R_1?

Ans.: To reduce the effect of offset voltage and thermal drift.

3) Balanced input amplifier, see Figure 8.6.

$$gain = \frac{R_f}{R_1}$$

$R_1 = R_2$

$R_f = R_3$

- Used when the connection to transducer pass through noisy environment.

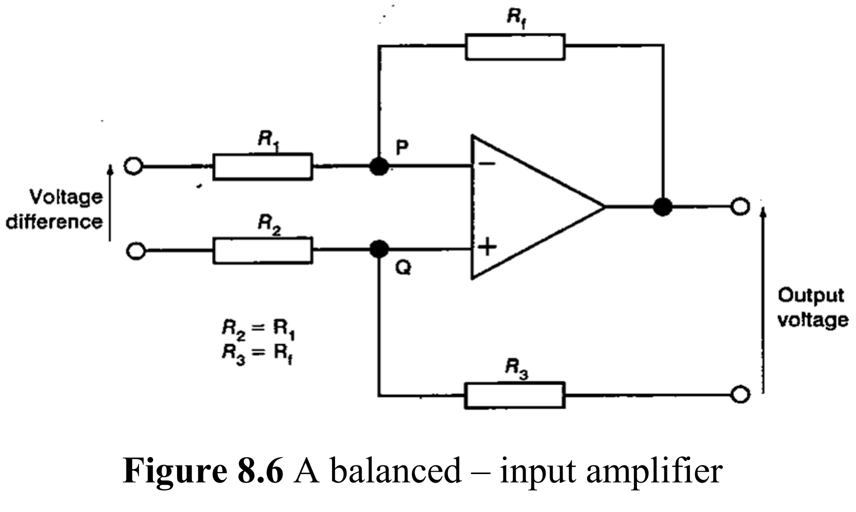


Figure 8.6 A balanced – input amplifier

4) Inverting summing amplifier, see Figure 8.7.

$i_1 + i_2 + \ldots = i_f$

$$\frac{V_{in}(1)}{R_1} + \frac{V_{in}(2)}{R_2} + \cdots = -\frac{V_o}{R_f}$$

$$V_{out} = -\left[\frac{R_f}{R_1}V_{in}(1) + \frac{R_f}{R_2}V_{in}(2) + \cdots\right]$$

$$gain = -\left[\frac{R_f}{R_1} + \frac{R_f}{R_2} + \frac{R_f}{R_3} + \cdots\right]$$

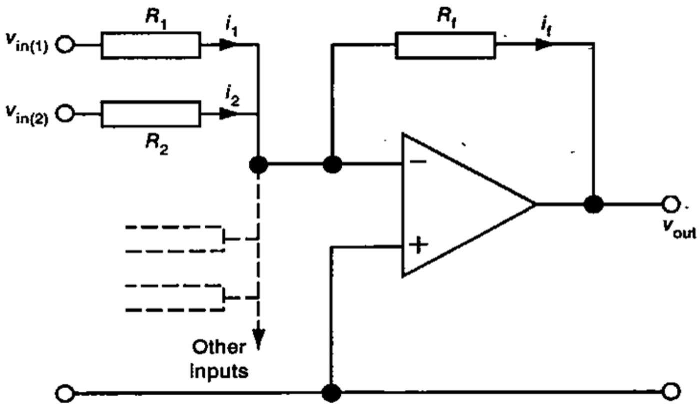


Figure 8.7 An inverting summing amplifier

5) Integrator, see Figure 8.8 and Figure 8.9.

- Reset switch used to make discharge for capacitor through (R_s) to start integral.

$$V_{out} = -\frac{1}{RC}\int V_{in}\, dt$$

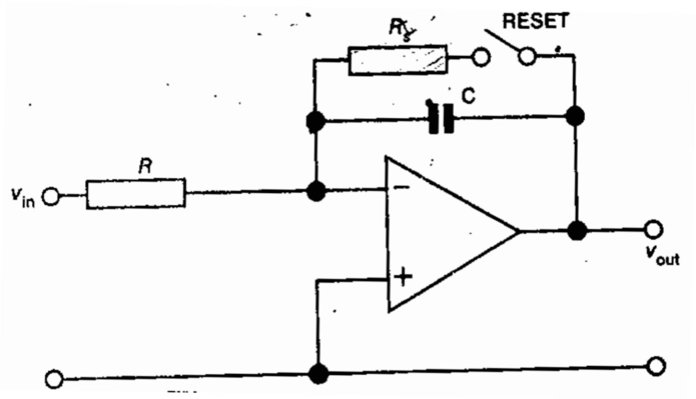


Figure 8.8 The op – amp as an integrator

Noise

Any unwanted alteration of the signal.

Q: List noise forms?

Ans.: **1)** Drift.

2) Internally generated random noise.

3) Noise picked up from external source.

Drift

Change in the gain of DC amplifier for DC signal voltage.

- Causes:

1) Variations in power supply voltage.

2) Temperature change with the amplifier.

3) Thermocouple.

- Solving

1) Use stabilized power supply.

2) Keep temperature constant by the use of heat sinks.

3) Temporarily short circuiting the input to the amplifier?

Q: How to convert DC output of transducer to AC?

Ans.: Chopper is transistor either:

1) In series with amplifier input as amplifier.

2) Parallel with it as short circuit.

Internally generated random noise:

Two main sources:

1) Thermal noise: come from electron movement due to thermal vibration of atoms of a conductor.

2) Shot noise: comes from random fluctuations in the passage of change carries through semiconductor material. (white noise).

- Solving:

1) Make signal-to-noise ratio high.

2) Preamplifier close to the transducer to improve signal-to-noise ratio.

External noise

1) Capacitive pick-up.
2) Inductive pick-up.

- Solving:

Use screening technique to reduce it.

Screening Technique

Enclosing cables, amplifiers in metal which is earthed so that the voltage picked up by the screening are short circuit to earth.

1) Capacitive pick-up screen: need metal have good conductor (copper).
2) Inductive pick-up screen: must be iron or other magnetic material.

Filters

- Use to exclude unwanted bandwidths from a signal.

Types:

1) Low – pass filter (exclude high frequency).
2) High – pass filter (exclude low frequency).
3) Band – pass filter (pass narrow frequency band).
4) Band – stop filter (exclude narrow band frequency).

See Figure 8.10.

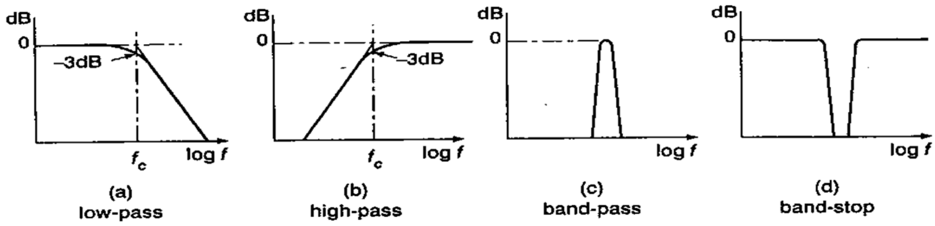


Figure 8.10 Typical frequency response curves of the main types of filter (a) low – pass, (b) high – pass, (c) band – pass, (d) band – stop

Filter Classification:

1) Passive
    - Use resistance, capacitance and/or inductance.
    - See Figure 8.11 a, b.

2) Active

- Use only op-amp with resistors and capacitors only.
- Need power supply.
- See Figure 8.11 c.

Passive	Active
1) More suitable for frequencies above audio range.	**1)** Better for low frequencies.
2) Simpler than active.	**2)** Small signal work and have gains greater than 0 dB.

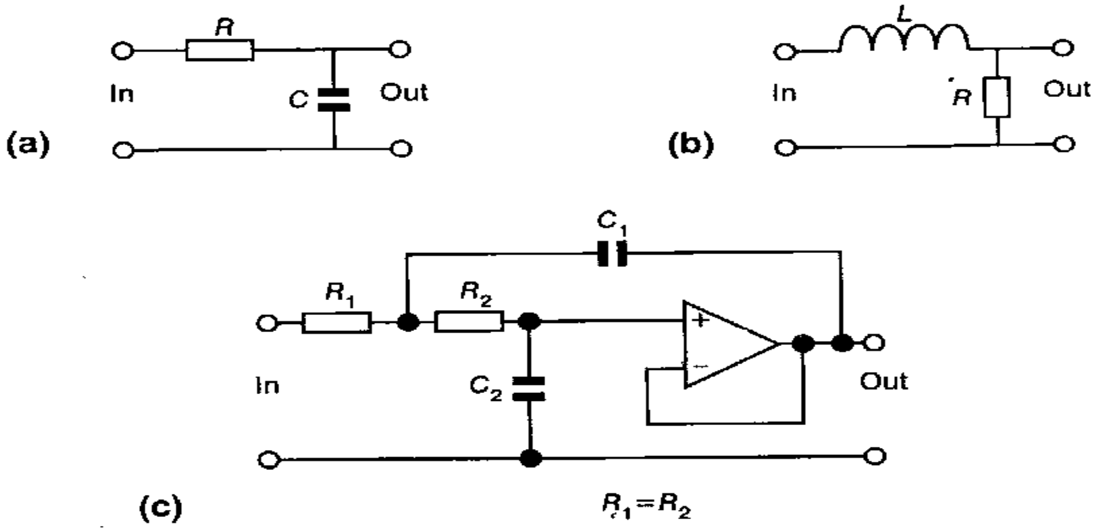


Figure 8.11 Low – pass filter circuits: (a) and (b) are first – order passive alternatives, (c) second – order active

3) Digital:

- Used with digital signals.

<u>Note:</u> these figures of passive filter for low – pass.

In case of high – pass change positions of (C) with (R) and (R) with (L) in passive, while in active change positions of (C) with (R) only.

Design Low and high pass:

$$f_c = \frac{1}{2\pi CR}$$

f_c : Corner frequency (Hz)

C: Capacitance in farad.

R: resistance in ohm.

Band pass filter: see Figure 8.12.

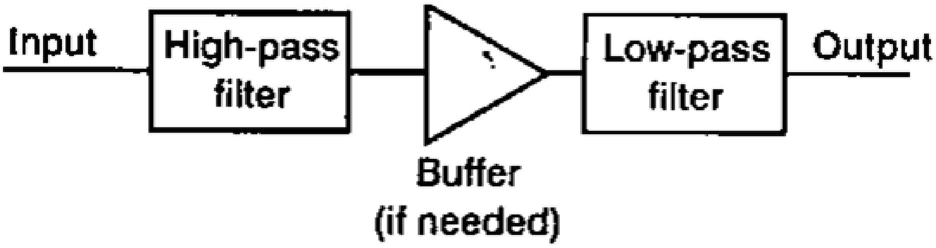


Figure 8.12 Band – pass filter

Band stop filter: see Figure 8.13

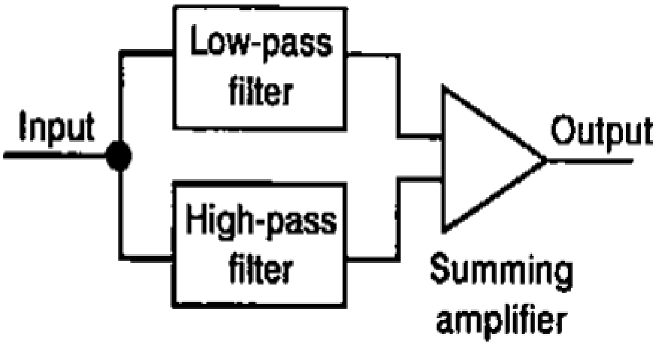


Figure 8.13 Band – stop filter

Notch filter: designed for remove one specific frequency from the signal rather than band.

Example 8.2

What amplitude will a centre-zero moving-coil meter show when an AC voltage of true amplitude 3 V is applied to it, if the frequency of the AC is such that the instrument's response is 3 dB down?

Solution

The amplitude of the pointer's motion diminishes as the frequency of the input AC is increased, because of damping and inertia of the pointer/moving-coil assembly. Because the scale is graduated in volts instead of degrees of rotation we can work directly in volts:

$$-3 = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

$$\therefore \log_{10}\left(\frac{V_2}{V_1}\right) = -0.15$$

$$\left(\frac{V_2}{V_1}\right) = 10^{-0.15}$$

$$= 0.708$$

$\therefore$ the meter indicates an amplitude of $0.708 \times 3\text{V} = 2.12$ V