Process Control

4th year. Control Engineering

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INTRODUCTION TO SENSOR

- What is Sensor?
  - Sensor converts the physical quantity to signal that can be recognized by other components such as display, transmitter and etc.

Physical Variable

(T, P, F, L, x, …)

Sensor

Measurement

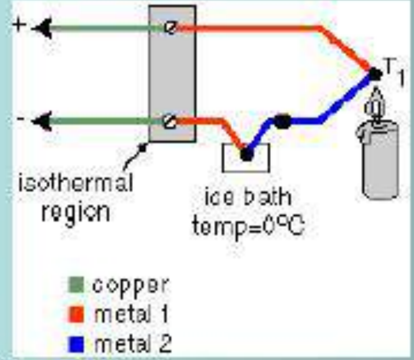
(V, mA, psig, …)

- Sensor types
  - Temperature: thermocouple, RTD, thermistor
  - Pressure: bellows, bourdon tube, diaphragm
  - Flow rate: orifice, venturi, magnetic, ultrasonic, Coliolis effect
  - Liquid level: float, differential pressure
  - pH: pH electrode
  - Viscosity: pressure drop across venturi or vane deflection
  - Composition: density, conductivity, GC, IR, NIR, UV

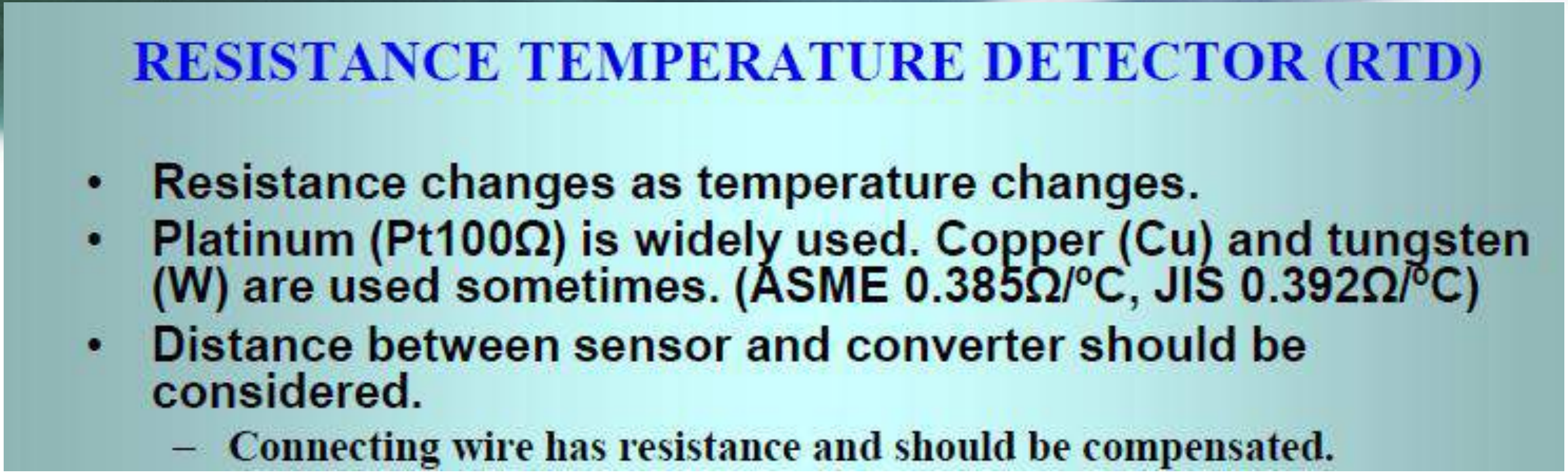
TEMPERATURE SENSORS

Principle	Type	Usable range (°C)	Remarks
Thermal Expansion	Gas expansion	-230~600	N2
	Liquid	-200~350	Oil
	Bimetal	-50~500	
Resistance	Pt-100	-200~500	Accurate, linear, self heating
	Thermistor	<300	Cheap, inaccurate, nonlinear
EMF	Thermocouple	-200~1600	Low sensitivity
	IC temp. sensor	-100~150	High voltage, accurate, linear
Radiation	Pyrometer	Very wide	Noncontacting, need accurate calibration

THERMOCOUPLE

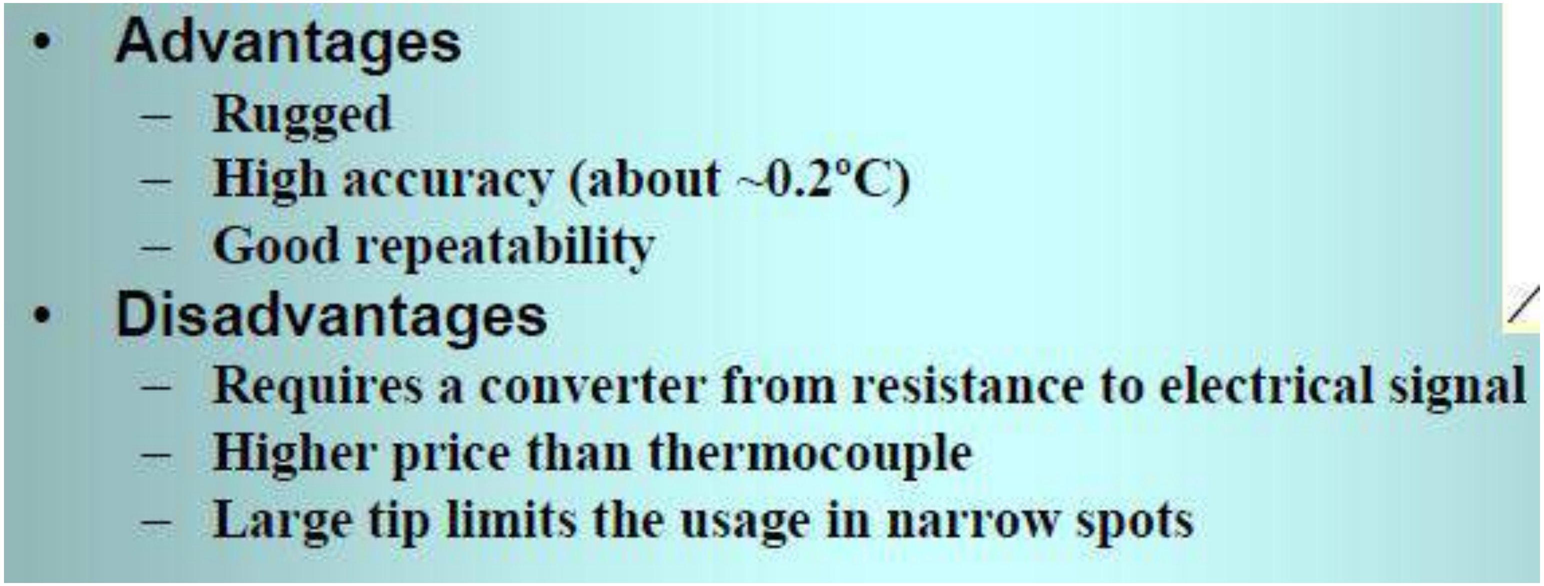


- Thermocouple Types
  - **Chromel- alumel (K- type): most popularly used**
  - **Iron- constantan (J- type): higher electromotive force (emf)**
  - **Chromel- constantan (E- Type): cryogenic temperature**
  - **13% Rh. Pt – Pt (R- type): high temperature (> 900°C)**
- Typical emf is about 0.041mV/°C for K type
  - **Needs signal amplification**
- Ice point can be a ice bath or an electronic device to compensate the ambient temperature.



RESISTANCE TEMPERATURE DETECTOR (RTD)

- Resistance changes as temperature changes.
- Platinum (Pt100Ω) is widely used. Copper (Cu) and tungsten (W) are used sometimes. (ASME 0.385Ω/°C, JIS 0.392Ω/°C)
- Distance between sensor and converter should be considered.
  - Connecting wire has resistance and should be compensated.



- Advantages
  - Rugged
  - High accuracy (about ~0.2°C)
  - Good repeatability
- Disadvantages
  - Requires a converter from resistance to electrical signal
  - Higher price than thermocouple
  - Large tip limits the usage in narrow spots

FLOW MEASUREMENT

- **Differential Pressure Cell**

$$Q = \frac{C_d A_2}{\sqrt{1-(A_2/A_1)^2}} \sqrt{\frac{2g_c\Delta P}{\rho}}$$

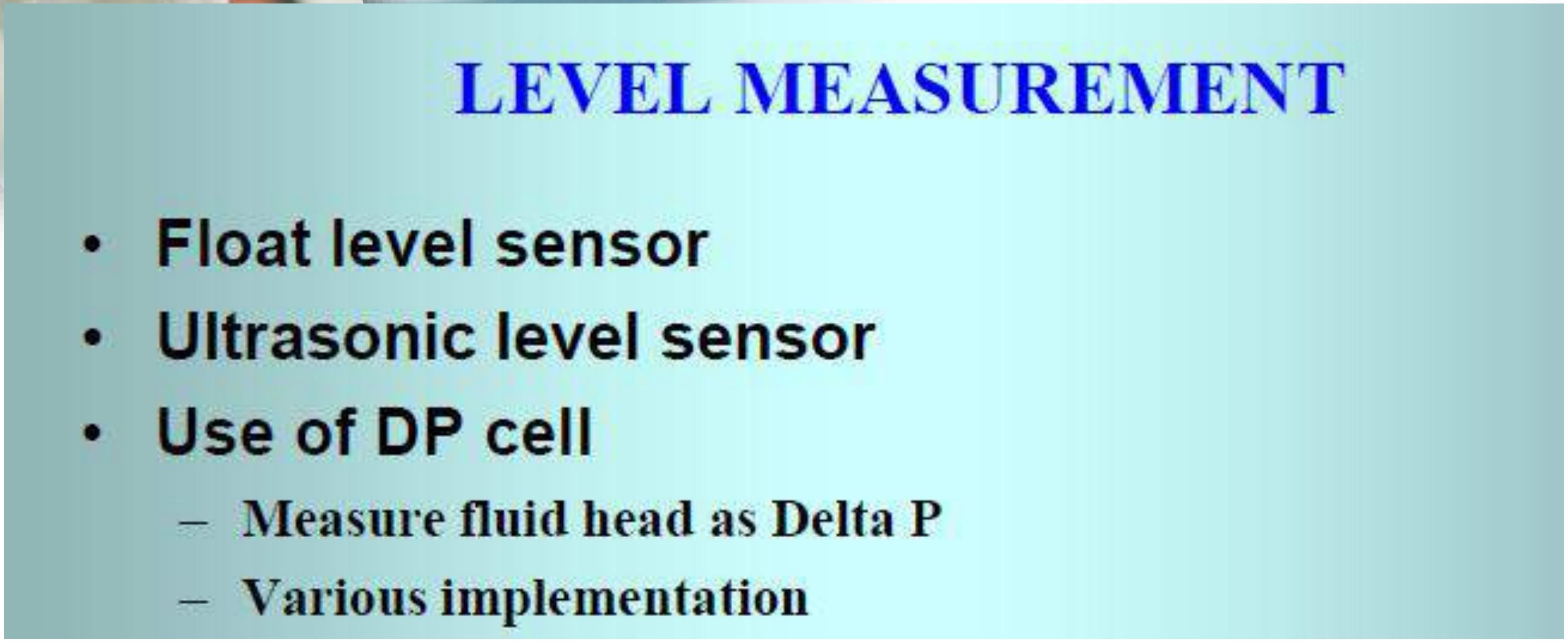
  - **ΔP: Delta P across the orifice**
  - **A_1: area of flow pipe**
  - **A_2: area of orifice**
  - **C_d: orifice coefficient**

– Maximum pressure drop should be < 4% of the total line pressure

– Selection of orifice size and delta P range is very important for the reading precision

FLOW MEASUREMENT(2)

- **Vortex Flow Meter**
  - The vortices create low and high pressure zones behind the bluff body.
  - The vortex meter uses a piezoelectric crystal sensor to detect the pressure exerted by the vortices on the sensing wing.
  - The piezoelectric crystal converts this vortex shedding frequency into electrical signals.
- **Electromagnetic Flow Meter**
  - Electrically conducting fluid passing through a magnetic field created by the device.

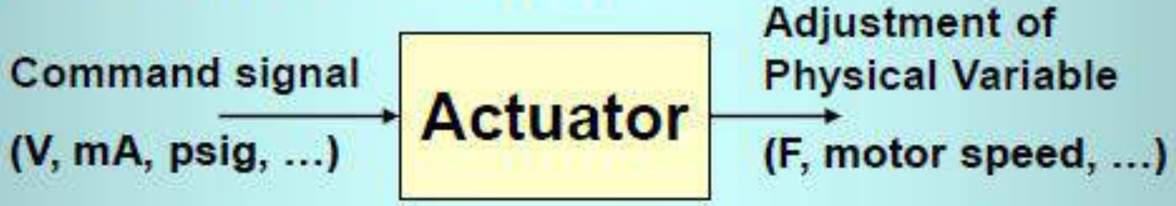


LEVEL MEASUREMENT

- Float level sensor
- Ultrasonic level sensor
- Use of DP cell
  - Measure fluid head as Delta P
  - Various implementation

INTRODUCTION TO ACTUATOR

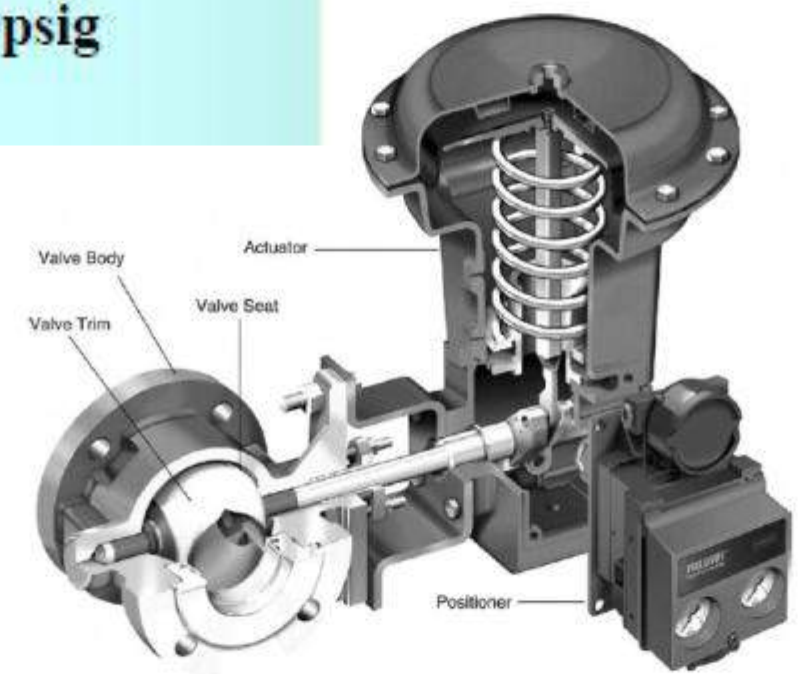
- **What is actuator?**
  - Actuator converts the command signal from controllers or higher-level components into physical adjustment in adjustable process variable



- **Actuator types**
  - Control valve: pneumatic, electric, hydraulic
  - Electric heater output: SCR, thyristor
  - Pump/Motor speed: inverter
  - Displacement: pneumatic, electric, hydraulic

CONTROL VALVE

- Valve+Actuator
  - Valve opening is adjusted by an actuator
- Pneumatic Control Valve
  - Usually 3~15psig signal is provided.
  - I/P transmitter converts 4~20mA signal to 3~15psig pneumatic signal via 20psig supply air

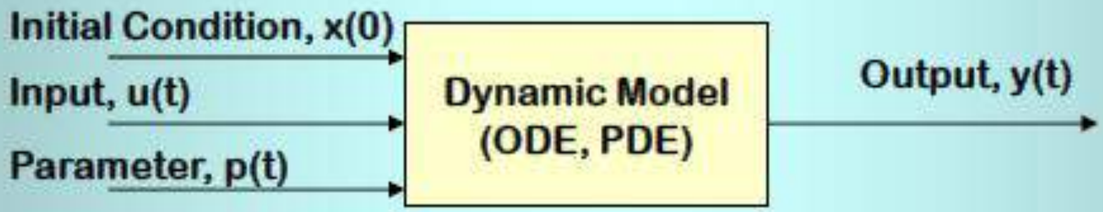


VALVE TRIM(PLUG) TYPE

- Equal Percentage
  - Most commonly used
  - Used where large pressure drop is expected
- Linear
  - Used where fairly constant press. drop is expected
  - Used for liquid level or flow loop
- Quick Opening
  - Used for frequently on-off service
  - Used where instantly large flow is needed

DYNAMIC VERSUS STEADY-STATE MODEL

- **Dynamic model**
  - Describes time behavior of a process
    - Changes in input, disturbance, parameters, initial condition, etc.
  - Described by a set of differential equations
    : ordinary (ODE), partial (PDE), differential-algebraic(DAE)



- **Steady-state model**
  - Steady state: No further changes in all variables
  - No dependency in time: No transient behavior
  - Can be obtained by setting the time derivative term zero

TYPICAL LINEAR DYNAMIC MODEL

- **Linear ODE**

$$\tau \frac{dy(t)}{dt} = -y(t) + Ku(t) \quad (\tau \text{ and K are contant, 1st order})$$

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \ldots + a_0 y(t)$$

$$= b_m \frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \ldots + b_0 u(t) \quad (\text{nth order})$$

- **Nonlinear ODE**

$$\tau \frac{dy(t)}{dt} = -y(t)^2 + Ku(t) \qquad \tau \frac{dy(t)}{dt} y(t) = -y(t)\sin(y) + Ku(t)$$

$$\tau \frac{dy(t)}{dt} = -y(t) + K\sqrt{u(t)} \qquad \tau \frac{dy(t)}{dt} = -e^{-y(t)} + Ku(t)$$

Useful references

- Lecture notes by Professor Dae Ryook Yang, Dept. of Chemical and Biological Engineering, Korea University,2006.
- Seborg, D. E., T. F. Edgar and D. A. Mellichamp (1989). *Process Dynamics and Control*, John Wiley & Sons, New York (U.S.A.)
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- Riggs, J. B. (1999). *Chemical Process Control*, Ferret Publishing, Lubbock (U.S.A.)

Thank you

Introduction to Process Control

Lecture 2, 2017/2018

Control & System Eng. Dept. , 4th year

Subject: Process Control.

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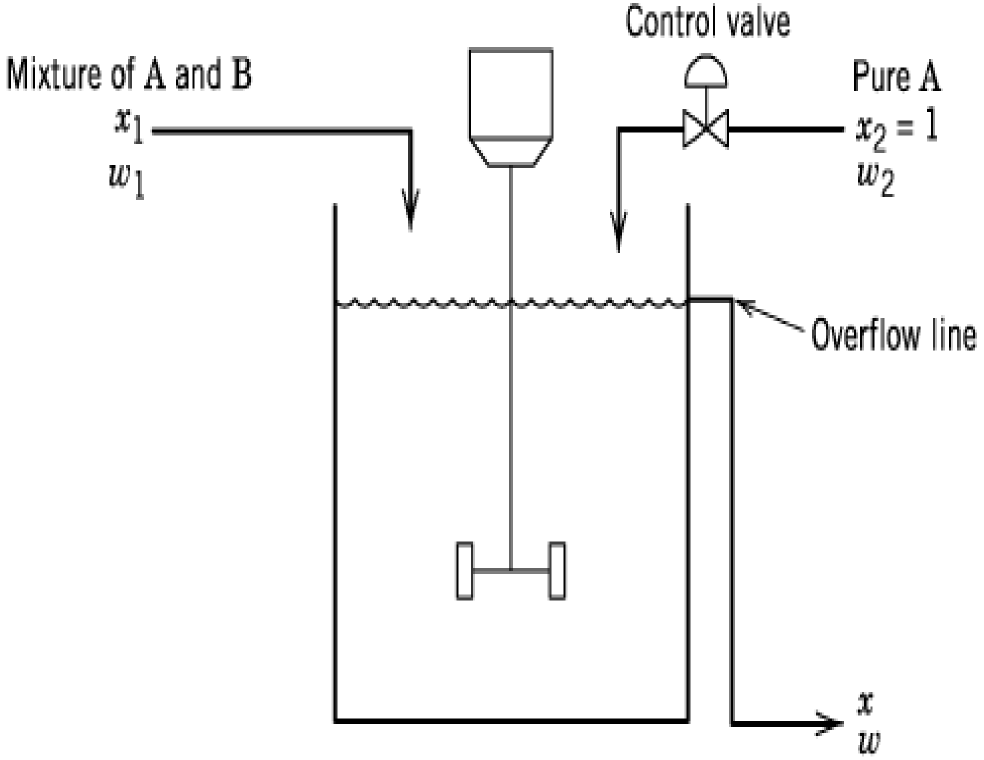
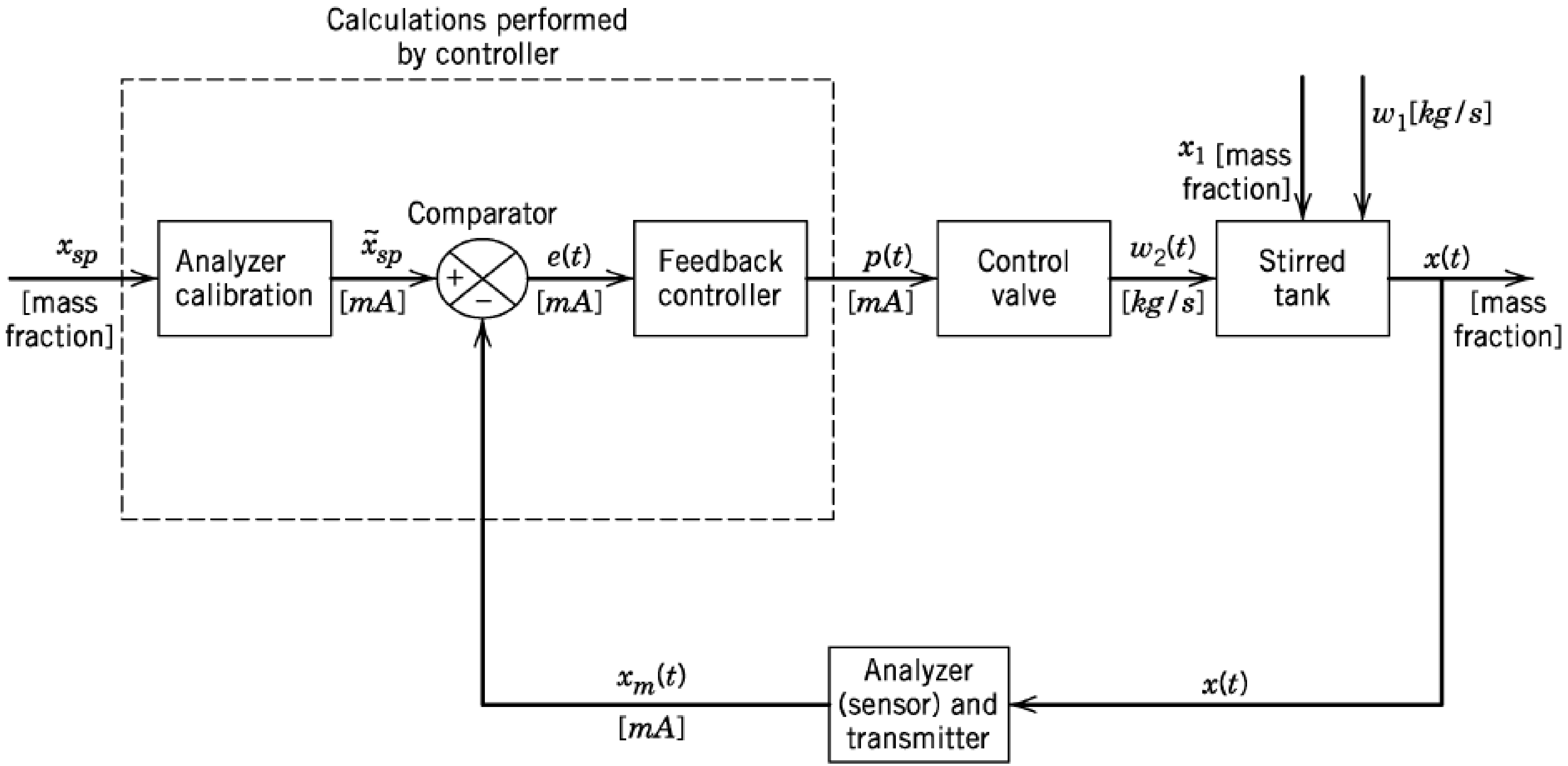
Process Control Diagrams

- consider the equipment that is used to implement control strategies.
- For the stirred-tank mixing system under feedback control.
- the exit concentration *x* is controlled and the flow rate W_2 of pure species A is adjusted using proportional control.



Block diagram for composition feedback control system



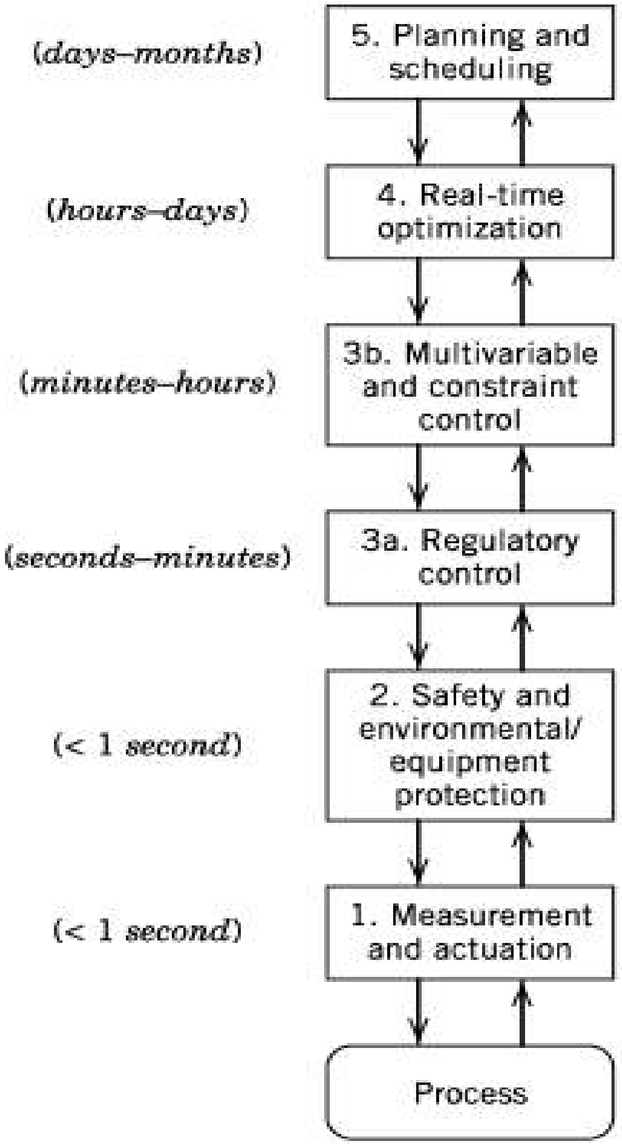
composition feedback control system

- Operation of the concentration control system can be summarized for the key hardware components as follows:
- **1. *Analyzer and transmitter*:** The tank exit concentration is measured by means of an instrument that generates a corresponding milliampere (mA)-level signal. This time-varying signal is then sent to the controller.
- **2. *Feedback controller*:** The controller performs three distinct calculations.
  - ***First***, it converts the actual set point X_{sp} into an equivalent internal signal $X_{sp}\cdot$
  - ***Second***, it calculates an error signal $e(t)$ by subtracting the measured value $Xm(t)$ from the set point isp' that is, $e(t) = X_{sp} - X_m(t).$
  - ***Third***, controller output $p(t)$ is calculated from the proportional control law.

composition feedback control system

- ***Control valve****:* The controller output *p(t)* in this case is a DC current signal that is sent to the control valve to adjust the valve stem position, which in turn affects flow rate $w_2(t)$.
- Because many control valves are pneumatic, i.e., are operated by air pressure, the controller output signal may have to be converted to an equivalent air pressure signal capable of adjusting the valve position.
- The block labeled "control valve" has *p(t)* as its input signal and $w_2(t)$ as its output signal, which illustrates that the signals on a block diagram can represent either a physical variable such as $w_2(t)$ or an instrument signal such as *p(t).*

THE HIERARCHY OF PROCESS CONTROL ACTIVITIES



THE HIERARCHY OF PROCESS CONTROL ACTIVITIES

- *Regulatory Control (**Level 3a**)*
  - *regulatory control,* is achieved by applying standard feedback and feedforward control techniques.
- *Multivariable and Constraint Control (**Level3b**)*
  - Many difficult process control problems have two distinguishing characteristics:
  - (i) significant interactions occur among key process variables,
  - and (ii) inequality constraints exist for manipulated and controlled variables.
- The ability to operate a process close to a limiting constraint is an important objective for advanced process control.

Theoretical Models of Chemical Processes

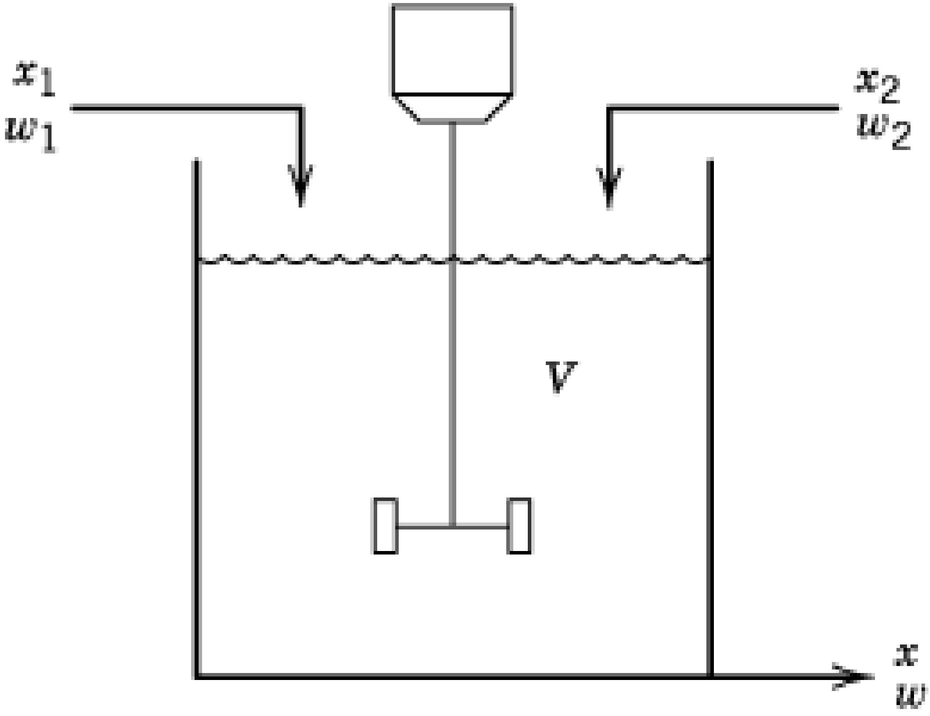
- Dynamic models play a central role in the subject of process dynamics and control. The models can be used to:
- 1. *Improve understanding of the process.*
- 2. *Train plant operating personnel.*
- 3. *Develop a control strategy for a new process.*
- 4. *Optimize process operating conditions.*

Theoretical Models of Chemical Processes

- Models can be classified based on how they are obtained:
- (a) *Theoretical models* are developed using the principles of chemistry, physics, and biology.
- (b) *Empirical models* are obtained by fitting experimental data.
- (c) *Semi-empirical models* are a combination of the models in categories (a) and (b);
- the numerical values of one or more of the parameters in a theoretical model are calculated from experimental data.

An Illustrative Example: A Blending Process

- consider the isothermal stirred-tank blending system
- the overflow line has been omitted and inlet stream 2 is not necessarily pure A (that is, $x2 \neq 1$).
- Now the volume of liquid in the tank V can vary with time,
- and the exit flow rate is not necessarily equal to the sum of the inlet flow rates.
- An unsteady-state mass balance for the blending system has the form



$$\left\{\begin{matrix}\text{rate of accumulation}\\ \text{of mass in the tank}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of}\\ \text{mass in}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of}\\ \text{mass out}\end{matrix}\right\}$$

- the rate of mass accumulation is simply *d(Vp)ldt,*
- where w_l, w_2, and w are mass flow rates.

$$\frac{d(V\rho)}{dt} = w_1 + w_2 - w$$

- We assume that the blending tank is perfectly mixed.
- This assumption has two important implications:
    - (i) there are no concentration gradients in the tank contents and
    - (ii) the composition of the exit stream is equal to the tank composition.
- The perfect mixing assumption is valid for low-viscosity liquids that receive an adequate degree of agitation.
- In contrast, the assumption is less likely to be valid for high-viscosity liquids such as polymers or molten metals.

- For the perfect mixing assumption, the rate of accumulation of component *A* is *d(Vpx)/dt,* where x is the mass fraction of *A.*
- The unsteady-state component balance is

$$\frac{d(V\rho x)}{dt} = w_1x_1 + w_2x_2 - wx$$

- *Question : how to find the steady state model ?*
- a steady-state model is a special case of an unsteady-state model that can be derived by setting accumulation terms equal to zero.

Conservation Laws

- Theoretical models of chemical processes are based on conservation laws such as the conservation of mass and energy.
- consider important conservation laws and use them to develop dynamic models for representative processes.
- ***Conservation of Mass***

$$\left\{\begin{array}{c}\text{rate of mass}\\ \text{accumulation}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\ \text{mass in}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{mass out}\end{array}\right\}$$

Conservation Laws

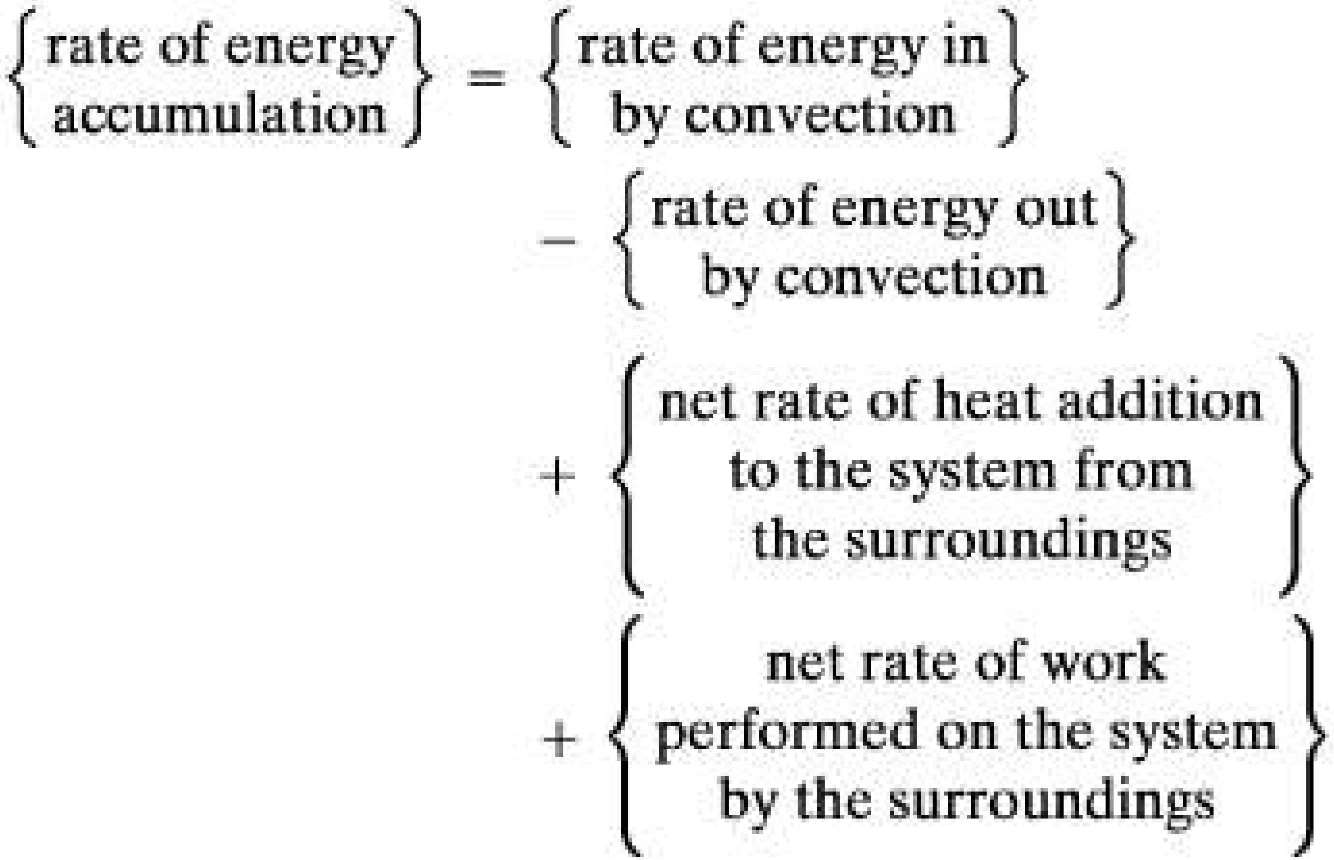
- *Conservation of Component i*

$$\begin{Bmatrix}\text{rate of component } i\\ \text{accumulation}\end{Bmatrix} = \begin{Bmatrix}\text{rate of component } i\\ \text{in}\end{Bmatrix} - \begin{Bmatrix}\text{rate of component } i\\ \text{out}\end{Bmatrix} + \begin{Bmatrix}\text{rate of component } i\\ \text{produced}\end{Bmatrix}$$

- Represents the rate of generation (or consumption) of component *i* as a result of chemical reactions.
- Conservation equations can also be written in terms of molar quantities, atomic species, and molecular species .

Conservation Laws

- ***Conservation of Energy***
- The general law of energy conservation is also called the First Law of Thermodynamics.



$$\left\{\begin{array}{c}\text{rate of energy}\\ \text{accumulation}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of energy in}\\ \text{by convection}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of energy out}\\ \text{by convection}\end{array}\right\} + \left\{\begin{array}{c}\text{net rate of heat addition}\\ \text{to the system from}\\ \text{the surroundings}\end{array}\right\} + \left\{\begin{array}{c}\text{net rate of work}\\ \text{performed on the system}\\ \text{by the surroundings}\end{array}\right\}$$

Conservation Laws

- The total energy of a thermodynamic system, U_{tot}, is the sum of its internal energy, kinetic energy, and potential energy:

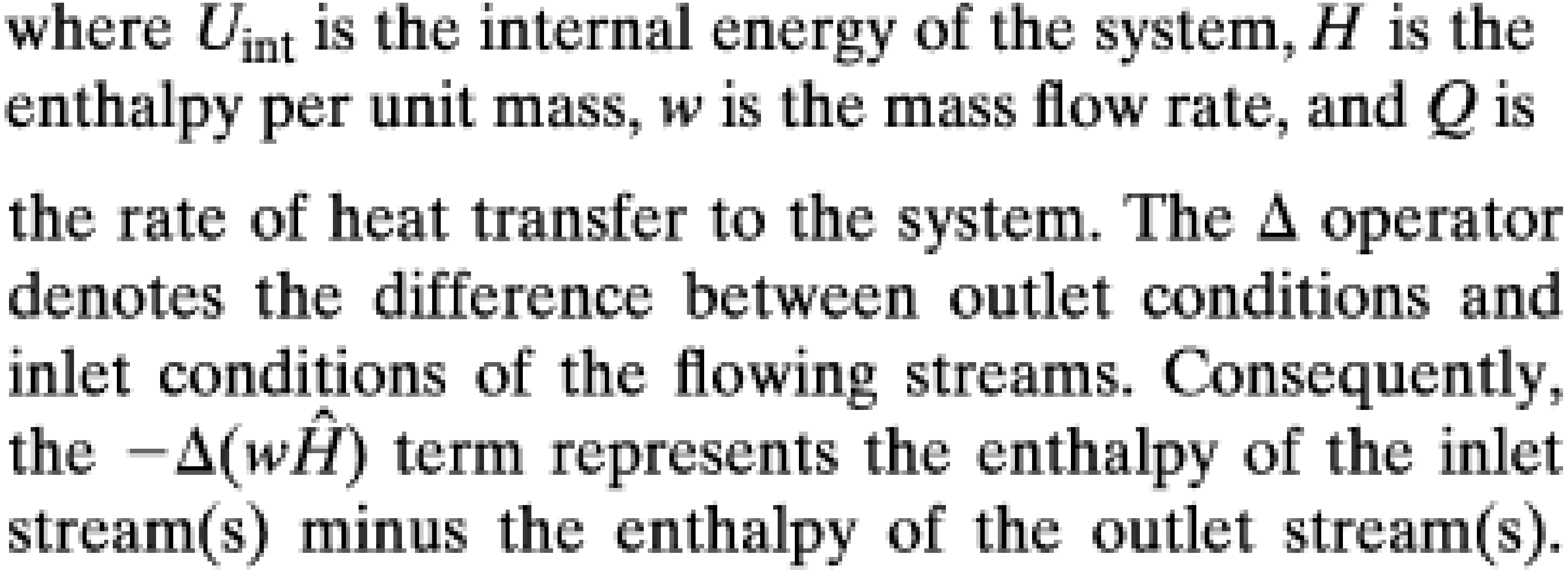
$$U_{\text{tot}} = U_{\text{int}} + U_{KE} + U_{PE}$$

- it is appropriate to make two assumptions:
- 1. Changes in potential energy and kinetic energy can be neglected, because they are small in comparison with changes in internal energy.
- 2. The net rate of work can be neglected, because it is small compared to the rates of heat transfer and convection.

Conservation Laws

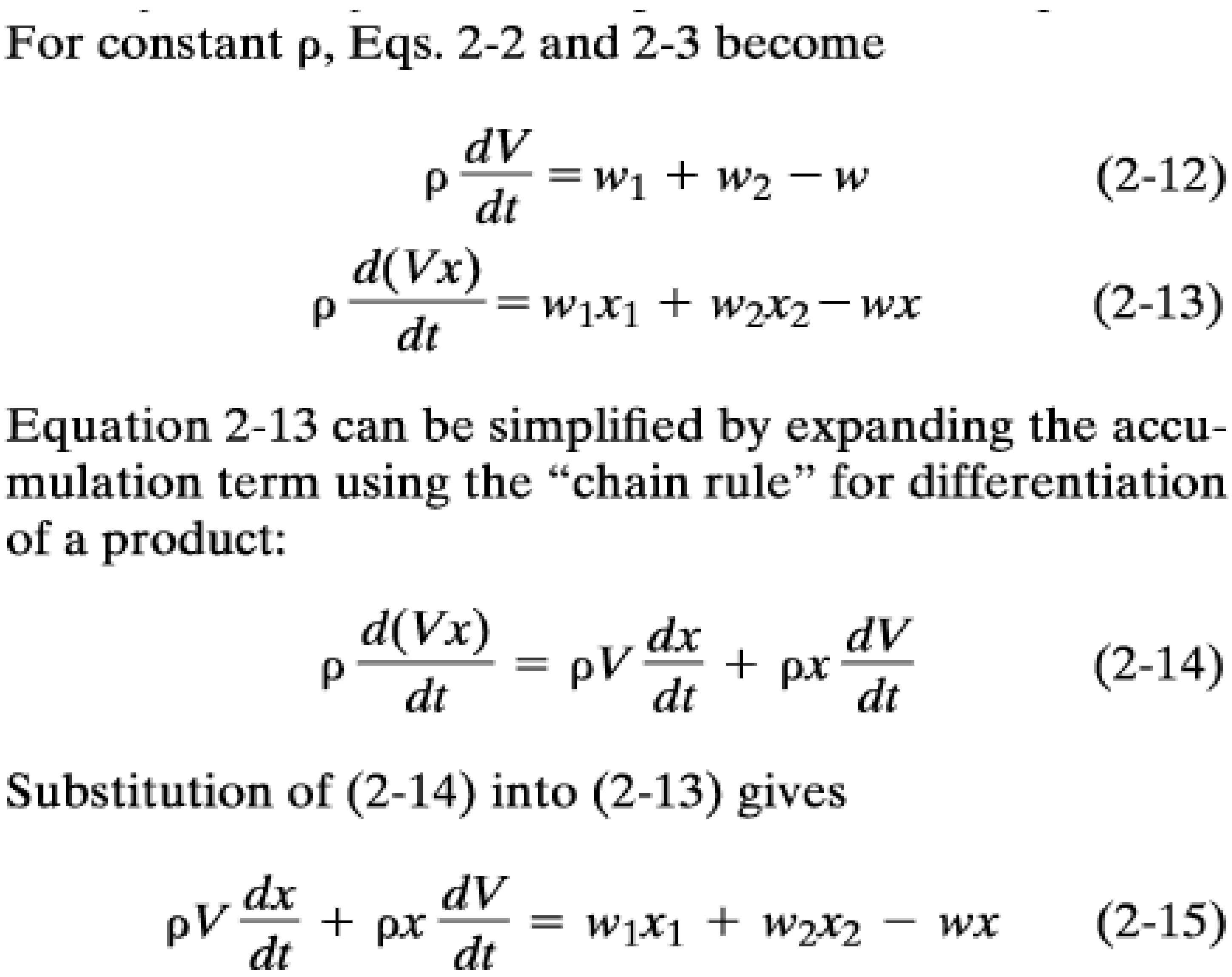
- Therefore, the energy balance can be written as:

$$\frac{dU_{\text{int}}}{dt} = -\Delta(w\hat{H}) + Q$$



where U_{int} is the internal energy of the system, $\hat{H}$ is the enthalpy per unit mass, w is the mass flow rate, and Q is the rate of heat transfer to the system. The Δ operator denotes the difference between outlet conditions and inlet conditions of the flowing streams. Consequently, the $-\Delta(w\hat{H})$ term represents the enthalpy of the inlet stream(s) minus the enthalpy of the outlet stream(s).

The Blending Process



For constant ρ, Eqs. 2-2 and 2-3 become

$$\rho \frac{dV}{dt} = w_1 + w_2 - w \tag{2-12}$$

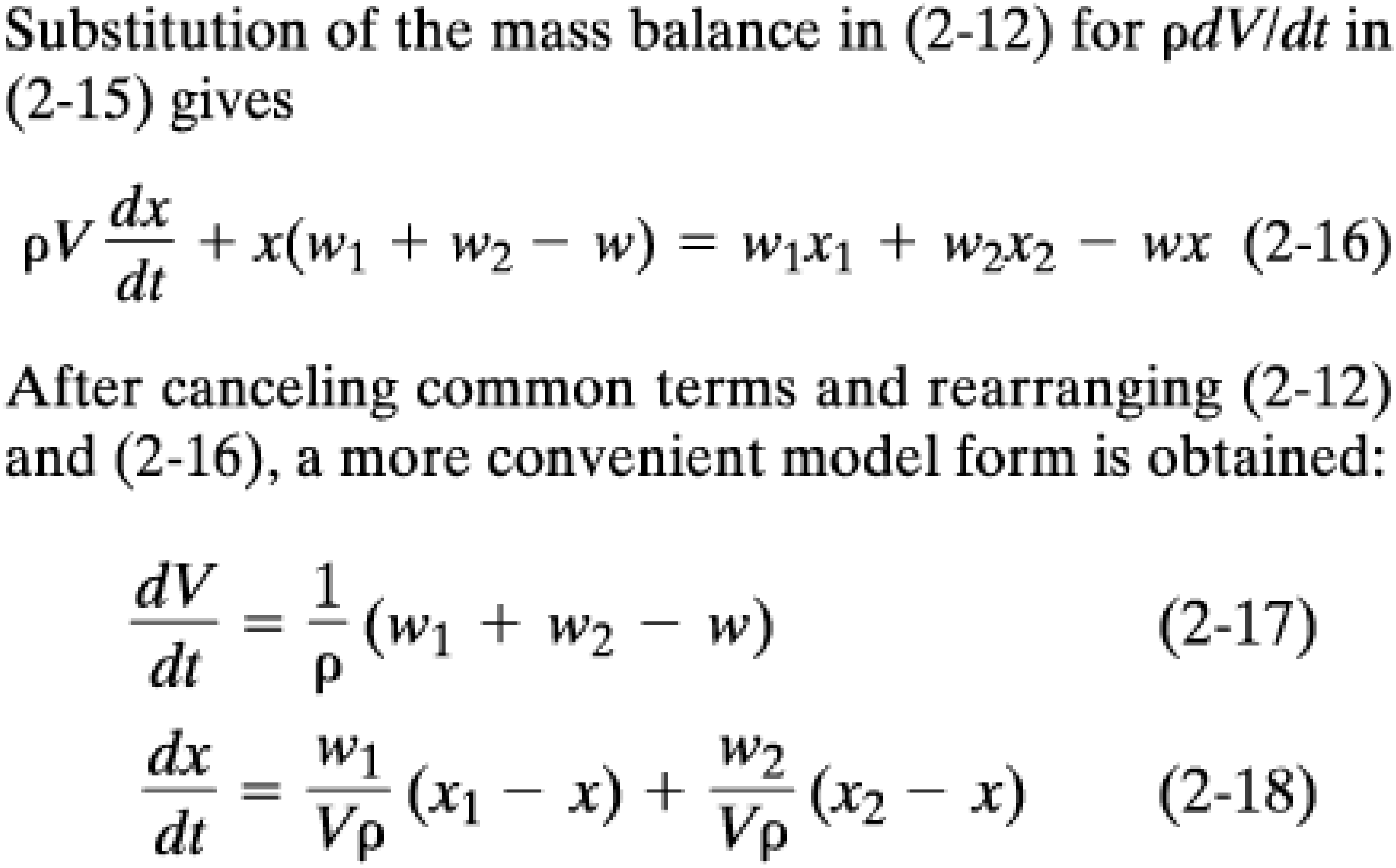
$$\rho \frac{d(Vx)}{dt} = w_1 x_1 + w_2 x_2 - wx \tag{2-13}$$

Equation 2-13 can be simplified by expanding the accumulation term using the “chain rule” for differentiation of a product:

$$\rho \frac{d(Vx)}{dt} = \rho V \frac{dx}{dt} + \rho x \frac{dV}{dt} \tag{2-14}$$

Substitution of (2-14) into (2-13) gives

$$\rho V \frac{dx}{dt} + \rho x \frac{dV}{dt} = w_1 x_1 + w_2 x_2 - wx \tag{2-15}$$



Substitution of the mass balance in (2-12) for $\rho dV/dt$ in (2-15) gives

$$\rho V \frac{dx}{dt} + x(w_1 + w_2 - w) = w_1 x_1 + w_2 x_2 - wx \quad (2\text{-}16)$$

After canceling common terms and rearranging (2-12) and (2-16), a more convenient model form is obtained:

$$\frac{dV}{dt} = \frac{1}{\rho}(w_1 + w_2 - w) \qquad (2\text{-}17)$$

$$\frac{dx}{dt} = \frac{w_1}{V\rho}(x_1 - x) + \frac{w_2}{V\rho}(x_2 - x) \qquad (2\text{-}18)$$

The dynamic model in Eqs. 2-17 and 2-18 is quite general and is based on only two assumptions: perfect mixing and constant density. For special situations, the liquid volume Vis constant (that is, $dV/dt = 0$), and the exit flow rate equals the sum of the inlet flow rates, $w = w1 + w2$

See Example 2.1

Reference:

Process Dynamics and Control, by Dale E. Seborg, Thomas F. Edgar, Duncan A. Mellichamp, Francis J. Doyle III, John Wiley & Sons, Inc.2011.

Process Control

Lecture 3

4th year. Control Engineering 2017-2018

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DEGREES OF FREEDOM ANALYSIS

- In order for the model to have a unique solution, the number of unknown variables must equal the number of independent model equations.
- An equivalent statement is that all of the available *degrees of freedom* must be utilized.
- The number of degrees of freedom, N_F can be calculated from the expression

$$N_F = N_V - N_E$$

- where N_V is the total number of process variables and N_E is the number of independent equations

DEGREES OF FREEDOM ANALYSIS

A degrees of freedom analysis allows modeling problems to be classified according to the following categories:

- **1.** $N_F = 0$: The process model is *exactly specified.* If $N_F = 0$, then the number of equations is equal to the number of process variables and the set of equations has a solution. (However, the solution may not be unique for a set of nonlinear equations.)
- 2. $N_F > 0$: The process is *underspecified.* If $N_F > 0$, then $N_V > N_E$, so there are more process variables than equations. Consequently, the N_E equations have an infinite number of solutions, because N_F process variables can be specified arbitrarily.
- 3. $N_F < 0$: The process model is *overspecified.* For $N_F < 0$, there are fewer process variables than equations, and consequently the set of equations has no solution.

Degrees of Freedom Analysis

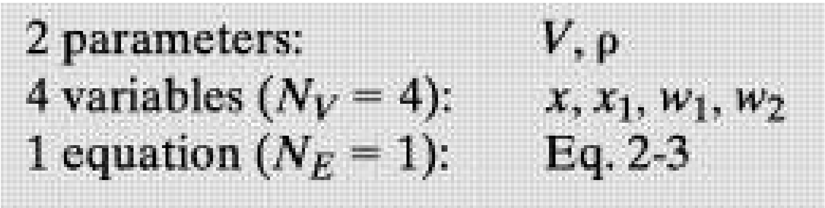
- 1. List all quantities in the model that are *known* constants (or parameters that can be specified) on the basis of equipment dimensions, known physical properties, and so on.
- 2. Determine the number of equations *N E* and the number of process variables, *Nv.* Note that time t is *not* considered to be a process variable, because it is neither a process input nor a process output.
- 3. Calculate the number of degrees of freedom, *NF= Nv- NE.*
- 4. Identify the *NE* output variables that will be obtained by solving the process model.
- 5. Identify the *NF* input variables that must be specified as either disturbance variables or manipulated variables, in order to utilize the *NF* degrees of freedom.

EXAMPLE

- Analyze the degrees of freedom for the blending model of Eq. (2-3) for the special condition where volume V is constant.

$$\frac{d(V\rho x)}{dt} = w_1x_1 + w_2x_2 - wx \qquad (2\text{-}3)$$

- ***SOLUTION***
- For this example, there are



2 parameters:	V, ρ
4 variables ($N_V = 4$):	x, x_1, w_1, w_2
1 equation ($N_E = 1$):	Eq. 2-3

- The degrees of freedom are calculated as *NF* = 4 - 1 = 3. Thus, we must identify three input variables that can be specified as known functions of time in order for the equation to have a unique solution. The dependent variable x is an obvious choice for the output variable in this simple example. Consequently, we have 1 output, 3 inputs.
- The three degrees of freedom can be utilized by specifying the inputs as
- 2 disturbance variables:
- 1 manipulated variable:
- Because all of the degrees of freedom have been utilized, the single equation is exactly specified and can be solved.

Example: Stirred-Tank Heating Process: Constant Holdup

- 2. The inlet and outlet flow rates are equal; thus, $w_i = w$ and the liquid holdup V is constant.
- 3. The density *p* and heat capacity *C* of the liquid are assumed to be constant. Thus, their temperature dependence is neglected.
- **4.** Heat losses are negligible.
- it is clear that we should consider an energy balance, because thermal effects predominate.
- A mass balance is not required in view of Assumptions 2 and 3.

- For a pure liquid at low or moderate pressures, the internal energy is approximately equal to the enthalpy $U_{\text{int}} \approx H$, and H depends only on temperature.
- we assume that $U_{\text{int}} = H$ and $\hat{U}_{\text{int}} = \hat{H}$, $(\hat{\ })$ means per unit mass.
- A differential change in temperature, *dT,* produces a corresponding change in the internal energy per unit mass, $d\hat{U}_{\text{int}}$,

$$d\hat{U}_{\text{int}} = d\hat{H} = C\,dT \qquad (2\text{-}29)$$

- where *C* is the constant pressure heat capacity (assumed to be constant). The total internal energy of the liquid in the tank can be expressed as the product of $\hat{U}_{\text{int}}$ and the mass in the tank, p V:

$$U_{\text{int}} = \rho V \hat{U}_{\text{int}} \qquad (2\text{-}30)$$

An expression for the rate of internal energy accumulation can be derived from Eqs. 2-29 and 2-30:

$$\frac{dU_{\text{int}}}{dt} = \rho VC \frac{dT}{dt} \qquad (2\text{-}31)$$

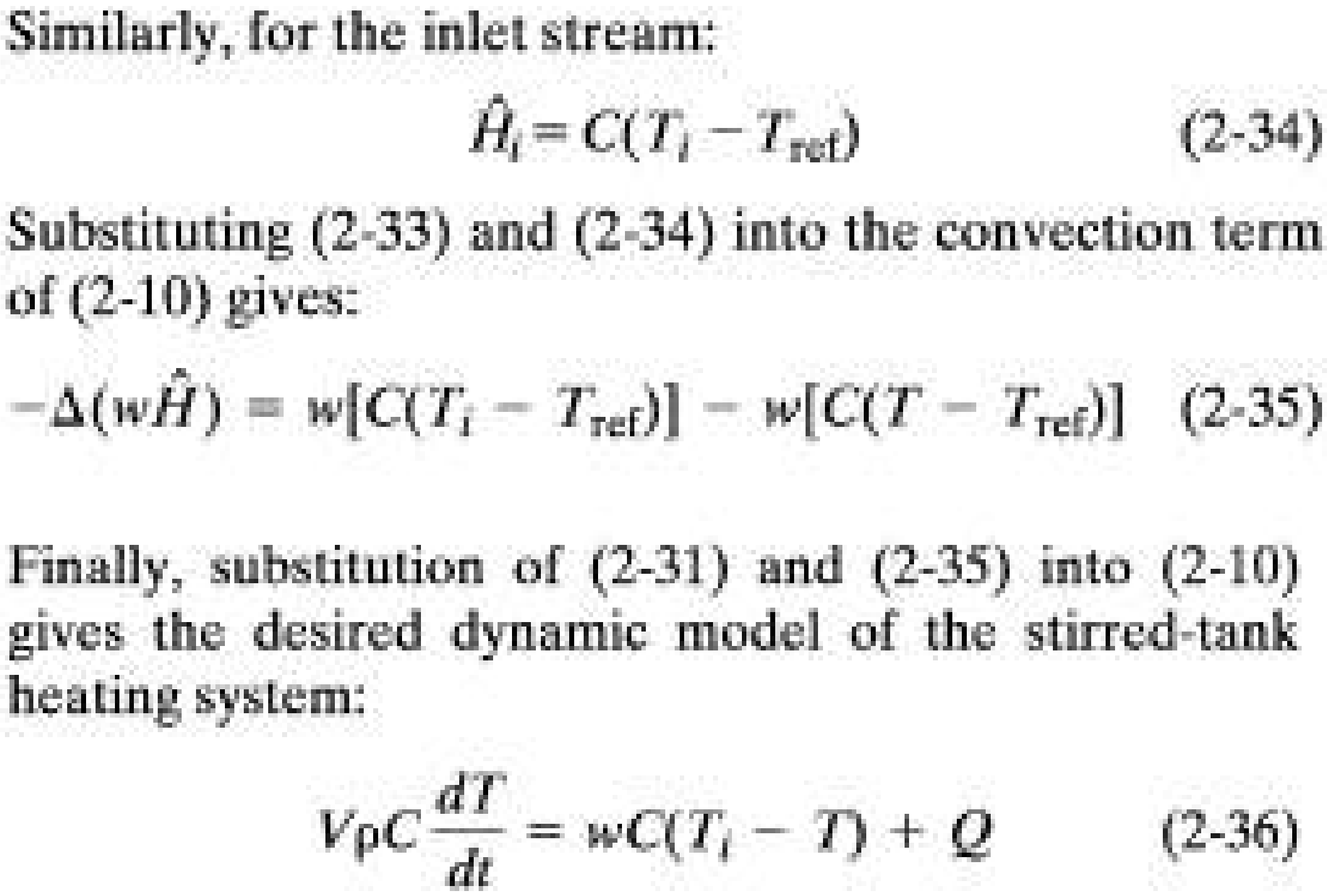
Note that this term appears in the general energy balance of Eq. 2-10.

Next, we derive an expression for the enthalpy term that appears on the right-hand side of Eq. 2-10. Suppose that the liquid in the tank is at a temperature T and has an enthalpy, $\hat{H}$. Integrating Eq. 2-29 from a reference temperature T_{ref} to T gives

$$\hat{H} - \hat{H}_{\text{ref}} = C(T - T_{\text{ref}}) \qquad (2\text{-}32)$$

where $\hat{H}_{\text{ref}}$ is the value of $\hat{H}$ at T_{ref}. Without loss of generality, we assume that $\hat{H}_{\text{ref}} = 0$
Thus, (2-32) can be written as:

$$\hat{H} = C(T - T_{\text{ref}}) \qquad (2\text{-}33)$$



Similarly, for the inlet stream:

$$\hat{H}_i = C(T_i - T_{ref}) \qquad (2\text{-}34)$$

Substituting (2-33) and (2-34) into the convection term of (2-10) gives:

$$-\Delta(w\hat{H}) = w[C(T_i - T_{ref})] - w[C(T - T_{ref})] \qquad (2\text{-}35)$$

Finally, substitution of (2-31) and (2-35) into (2-10) gives the desired dynamic model of the stirred-tank heating system:

$$V\rho C \frac{dT}{dt} = wC(T_i - T) + Q \qquad (2\text{-}36)$$

Note that the Tref terms have canceled, because *C* was assumed to be constant, and thus independent of temperature.

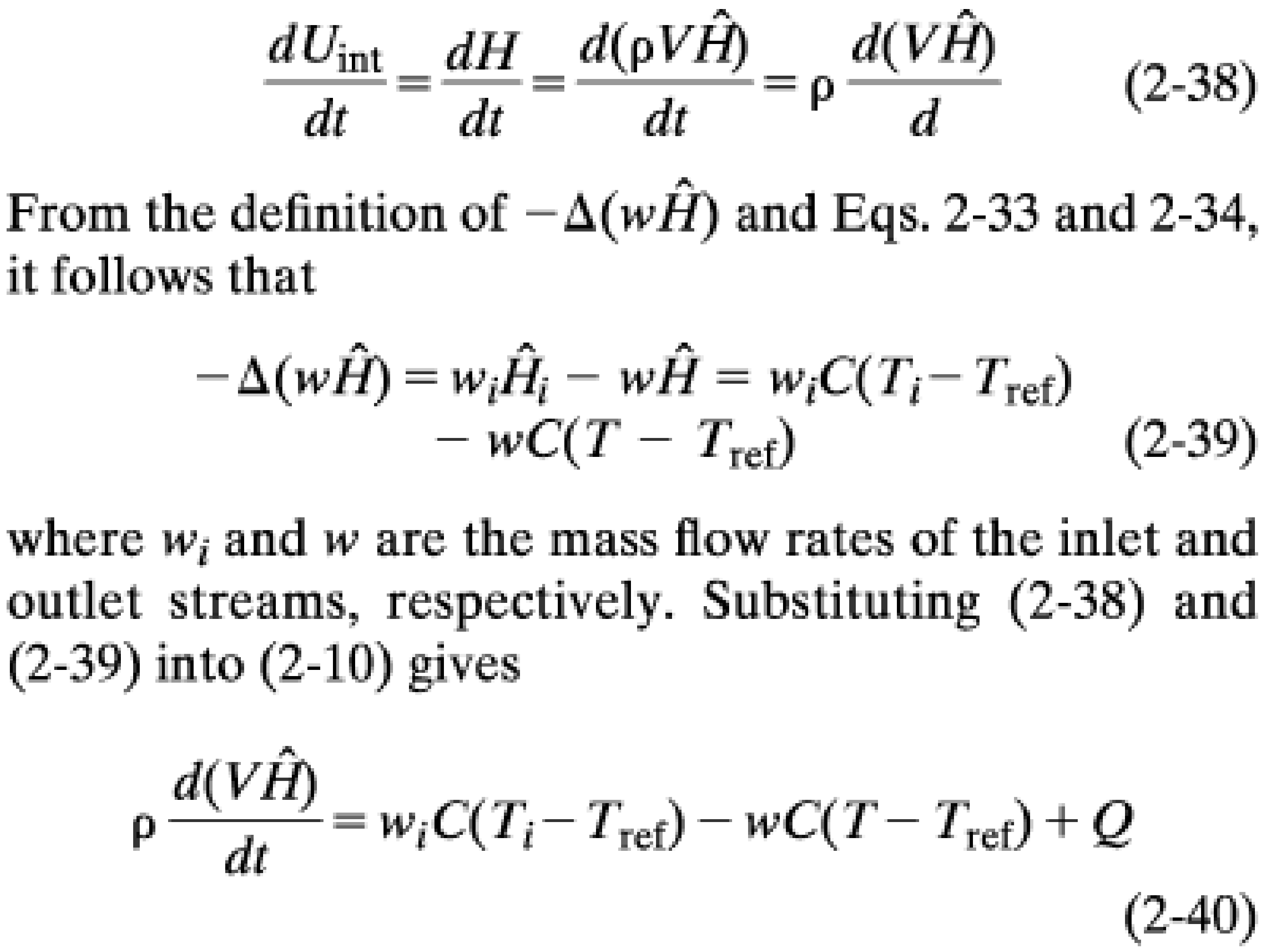
Example:
Stirred-Tank Heating Process: Variable Holdup

- consider the more general situation in which the tank holdup can vary with time. This analysis also is based on assumptions 1, 3 and 4 of the previous section.
- Now an overall mass balance is needed, because the holdup is not constant.
- The overall mass balance is

$$\frac{d(V\rho)}{dt} = w_i - w \qquad (2\text{-}37)$$

- The energy balance for the current stirred-tank heating system can be derived from Eq. 2-10 in analogy with the derivation of Eq. 2-36.
- We again assume that $U_{int} = H$ for the liquid in the tank.

- Thus, for constant *p*:



$$\frac{dU_{\text{int}}}{dt} = \frac{dH}{dt} = \frac{d(\rho V\hat{H})}{dt} = \rho\,\frac{d(V\hat{H})}{d} \qquad (2\text{-}38)$$

From the definition of $-\Delta(w\hat{H})$ and Eqs. 2-33 and 2-34, it follows that

$$\begin{aligned} -\Delta(w\hat{H}) = w_i\hat{H}_i - w\hat{H} &= w_iC(T_i - T_{\text{ref}}) \\ &\quad - wC(T - T_{\text{ref}}) \end{aligned} \qquad (2\text{-}39)$$

where w_i and w are the mass flow rates of the inlet and outlet streams, respectively. Substituting (2-38) and (2-39) into (2-10) gives

$$\rho\,\frac{d(V\hat{H})}{dt} = w_iC(T_i - T_{\text{ref}}) - wC(T - T_{\text{ref}}) + Q \qquad (2\text{-}40)$$

Next we simplify the dynamic model. Because ρ is constant, (2-37) can be written as

$$\rho \frac{dV}{dt} = w_i - w \tag{2-41}$$

The chain rule can be applied to expand the left side of (2-40) for constant C and ρ:

$$\rho \frac{d(V\hat{H})}{dt} = \rho V \frac{d\hat{H}}{dt} + \rho \hat{H} \frac{dV}{dt} \tag{2-42}$$

From Eq. 2-29 or 2-33, it follows that $d\hat{H}/dt = CdT/dt$. Substituting this expression and Eqs. 2-33 and 2-41 into Eq. 2-42 gives

$$\rho \frac{d(V\hat{H})}{dt} = C(T - T_{\text{ref}})(w_i - w) + \rho CV \frac{dT}{dt} \tag{2-43}$$

Substituting (2-43) into (2-40) and rearranging gives

$$C(T - T_{\text{ref}})(w_i - w) + \rho C V \frac{dT}{dt} = w_i C(T_i - T_{\text{ref}}) - wC(T - T_{\text{ref}}) + Q \quad (2\text{-}44)$$

Rearranging (2-41) and (2-44) provides a simpler form for the dynamic model:

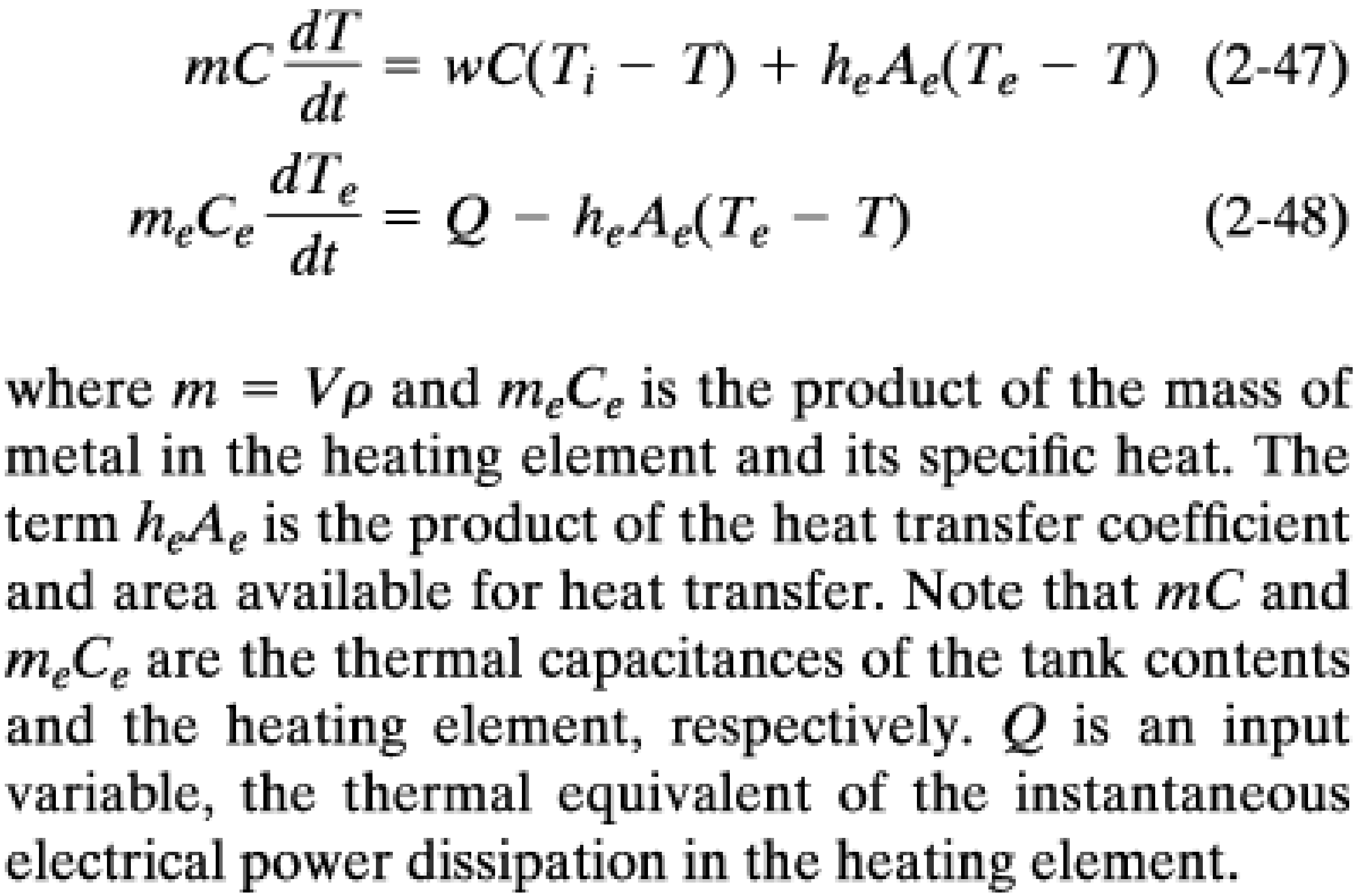
$$\frac{dV}{dt} = \frac{1}{\rho}(w_i - w) \quad (2\text{-}45)$$

$$\frac{dT}{dt} = \frac{w_i}{V\rho}(T_i - T) + \frac{Q}{\rho C V} \quad (2\text{-}46)$$

Electrically Heated Stirred Tank

- consider the stirred-tank heating system with constant holdup (Example 1) but we relax the
- assumption that energy is transferred instantaneously from the heating element to the contents of the tank.
- Suppose that the metal heating element has a significant thermal capacitance and that the electrical heating rate *Q* directly affects the temperature of the element rather than the liquid contents.
- For simplicity, we neglect the temperature gradients in the heating element that result from heat conduction and assume that the element has a uniform temperature, *Te.*
- This temperature can be interpreted as the average temperature for the heating element.

- Based on this new assumption, and the previous assumptions , the unsteady-state energy balances for the tank and the heating element can be written as



$$mC\frac{dT}{dt} = wC(T_i - T) + h_eA_e(T_e - T) \quad (2\text{-}47)$$

$$m_eC_e\frac{dT_e}{dt} = Q - h_eA_e(T_e - T) \quad (2\text{-}48)$$

where $m = V\rho$ and m_eC_e is the product of the mass of metal in the heating element and its specific heat. The term h_eA_e is the product of the heat transfer coefficient and area available for heat transfer. Note that mC and m_eC_e are the thermal capacitances of the tank contents and the heating element, respectively. Q is an input variable, the thermal equivalent of the instantaneous electrical power dissipation in the heating element.

If flow rate w is constant, Eqs. 2-47 and 2-48 can be converted into a single second-order differential equation. First, solve Eq. 2-47 for T_e and then differentiate to find dT_e/dt. Substituting the expressions for T_e and dT_e/dt into Eq. 2-48 yields

$$\frac{mm_eC_e}{wh_eA_e}\frac{d^2T}{dt^2} + \left(\frac{m_eC_e}{h_eA_e} + \frac{m_eC_e}{wC} + \frac{m}{w}\right)\frac{dT}{dt} + T = \frac{m_eC_e}{h_eA_e}\frac{dT_i}{dt} + T_i + \frac{1}{wC}Q \qquad (2\text{-}49)$$

See example 2.4

- Reference:
- Process Dynamics and Control, by Dale E. Seborg, Thomas F. Edgar, Duncan A. Mellichamp, Francis J. Doyle III, John Wiley & Sons, Inc.2011.

Introduction to Process Control

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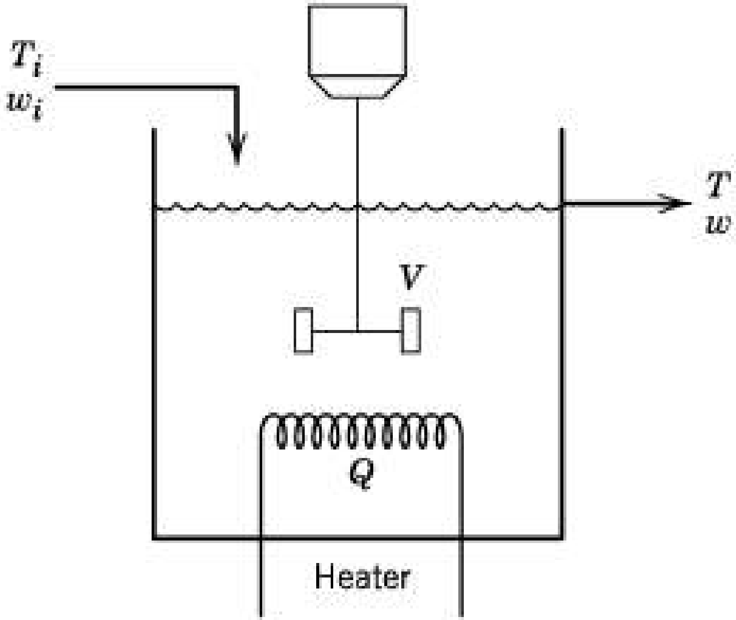
Steam-Heated Stirred Tank

- Steam (or some other heating medium) can be condensed within a coil or jacket to heat liquid in a stirred tank, and the inlet steam pressure can be varied by adjusting a control valve.
- The condensation pressure *Ps* then fixes the steam temperature *Ts* through an appropriate thermodynamic relation or from tabular information such as the steam tables

$$T_s = f(P_s)$$

- Consider the stirred-tank heating system with constant holdup and a steam heating coil.
- We assume that the thermal capacitance of the liqui condensate is negligible compared to the thermal capacitances of the tank liquid and the wall of the heating coil.
- This assumption is reasonable when a steam trap is used to remove the condensate from the coil as it is produced.
- As a result of this assumption, the dynamic model consists of energy balances on the liquid and the heating coil wall:



$$mC\frac{dT}{dt} = wC(T_i - T) + h_pA_p(T_w - T) \qquad (2\text{-}51)$$

$$m_wC_w\frac{dT_w}{dt} = h_sA_s(T_s - T_w) - h_pA_p(T_w - T) \qquad (2\text{-}52)$$

- where the subscripts *w,* s, and *p* refer, respectively, to the wall of the heating coil and to its steam and process sides.
- Note that these energy balances are similar to Eqs. 2-47 and 2-48 for the electrically heated example.
- The dynamic model contains three output variables *(Ts, T,* and *Tw)* and three equations: an algebraic equation with *Ts* related to *Ps* (a specified function of time or a constant) and two differential equations. Thus,
- Eqs. 2-50 through 2-52 constitute an exactly specified model with three input variables: *Ps,* T_i*,* and *w.*
- Several important features are noted.

- 1. Usually *hsAs* >> *hp-Ap,* because the resistance to heat transfer on the steam side of the coil is much
- lower than on the process side.
- **2.** The change from electrical heating to steam heating increases the complexity of the model (three equations instead of two) but does not increase the model order (number of first-order differential equations).
- **3.** As models become more complicated, the input and output variables may be coupled through certain parameters.
- For example, *hp* may be a function of *w,* or *hs* may vary with the steam condensation rate; sometimes algebraic equations cannot be
- solved explicitly for a key variable.
- In this situation, numerical solution techniques have to be used.
- Usually, implicit algebraic equations must be solved by iterative methods at each time step in the numerical integration.

Liquid Storage Systems

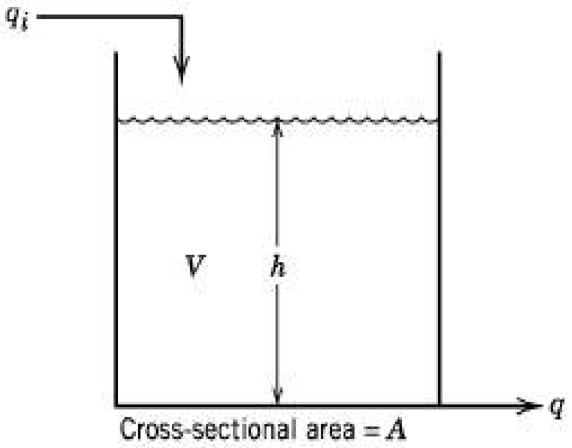
- A typical liquid storage process is shown in Fig. 2.5 where *q;* and *q* are volumetric flow rates. A mass balance yields

$$\frac{d(\rho V)}{dt} = \rho q_i - \rho q \qquad (2\text{-}53)$$

- Assume that liquid density p is constant and the tank is cylindrical with cross-sectional area, *A.*
- Then the volume of liquid in the tank can be expressed as *V* = *Ah,* where h is the liquid level (or *head).* Thus, (2-53) becomes

$$A\frac{dh}{dt} = q_i - q \qquad (2\text{-}54)$$

- Note that Eq. 2-54 appears to be a *volume balance.*
- There are three important variations of the liquid storage process:
- **1.** The inlet or outlet flow rates might be constant;
- for example, exit flow rate *q* might be kept constant by a constant-speed, fixed-volume (metering) pump.
- An important consequence of this configuration is that the exit flow rate is then completely independent of liquid level over a wide range of conditions.
- Consequently, $q = \overline{q}$ where $\overline{q}$
- is the steady-state value.
- For this situation, the tank operates essentially as a flow *integrator*



- The tank exit line may function simply as a resistance to flow from the tank (distributed along the entire line), or it may contain a valve that provides significant resistance to flow at a single point.
- In the simplest case, the flow may be assumed to be linearly related to the driving force, the liquid level, in analogy to Ohm's law for electrical circuits *(E = IR)*

$$h = qR_v \tag{2-55}$$

- where *Rv* is the resistance of the line or valve.
- Rearranging (2-55) gives the following *flow-head equation:*

$$q = \frac{1}{R_v} h \tag{2-56}$$

- Substituting (2-56) into (2-54) gives a first-order differential equation:

$$A\frac{dh}{dt} = q_i - \frac{1}{R_v}h \qquad (2\text{-}57)$$

- This model of the liquid storage system exhibits dynamic behavior similar to that of the stirred tank heating system of Eq. 2-36.
- **3**. A more realistic expression for flow rate *q* can be obtained when a fixed valve has been placed in the exit line and turbulent flow can be assumed.
- The driving force for flow through the valve is the pressure drop ΔP:

$$\Delta P = P - P_a \qquad (2\text{-}58)$$

- where *P* is the pressure at the bottom of the tank and *Pa is* the pressure at the end of the exit line.
- We assume that *Pa is* the ambient pressure.
- If the valve is considered to be an orifice, a mechanical energy balance, or *Bernoulli equation* , can be used to derive the relation

$$q = C_v^* \sqrt{\frac{P - P_a}{\rho}} \qquad (2\text{-}59)$$

- where *Cv** is a constant. The value of *Cv** depends on the particular valve and the valve setting (how much it is open).
- The pressure P at the bottom of the tank is related to liquid level *h* by a force balance

$$P = P_a + \frac{\rho g}{g_c} h \qquad (2\text{-}60)$$

- where the acceleration of gravity *g* is constant.
- Substituting (2-59) and (2-60) into (2-54) yields the dynamic model

$$A\frac{dh}{dt} = q_i - C_v\sqrt{h} \qquad (2\text{-}61)$$

where $C_v \overset{\Delta}{=} C_v^*\sqrt{g/g_c}$. This model is nonlinear due to the square root term.

The Continuous Stirred-Tank Reactor (CSTR)

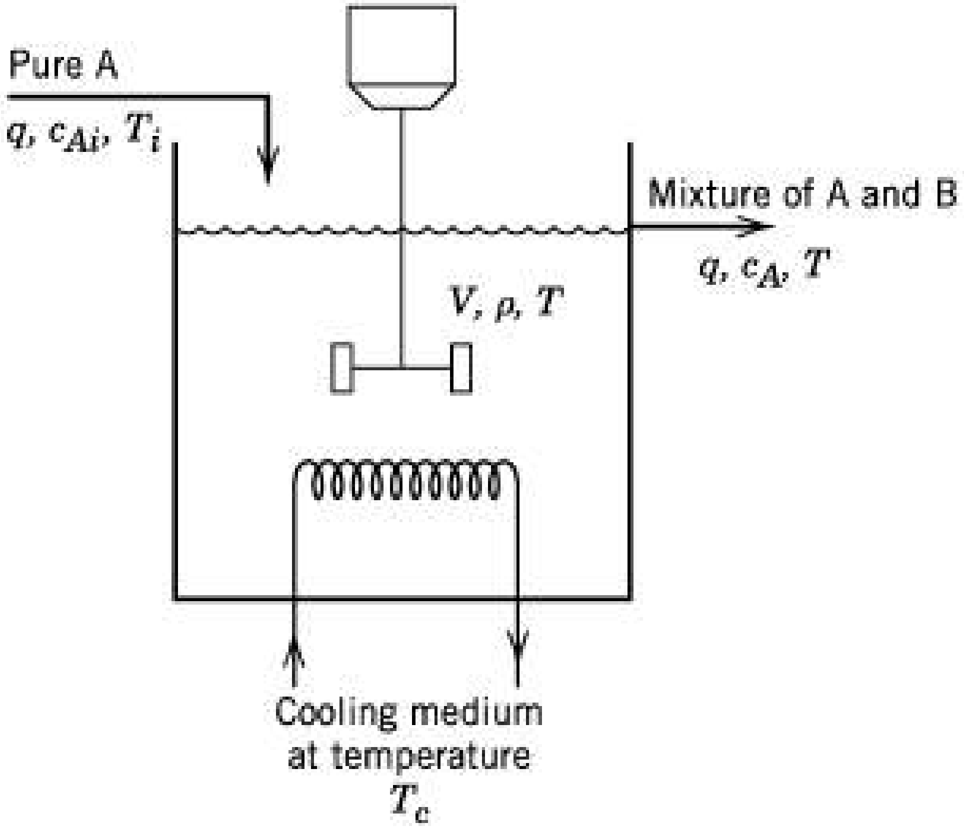
- Consider a simple liquid-phase, irreversible chemical reaction where chemical species A reacts to form species B. The reaction can be written as A→ B.
- We assume that the rate of reaction is first-order with respect to component A,

$$r = kc_A \tag{2-62}$$

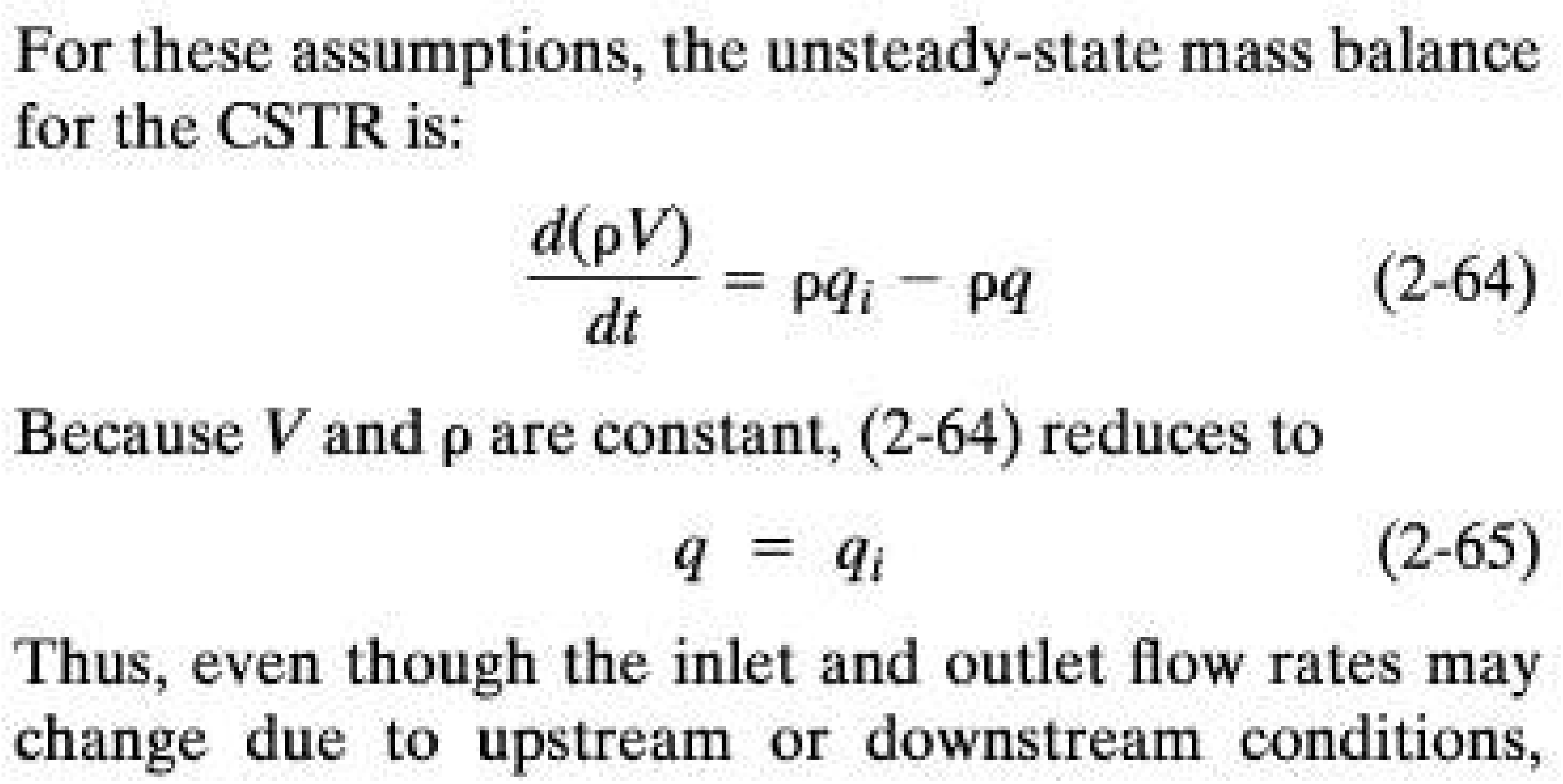
- where *r* is the rate of reaction of A per unit volume, *k* is the reaction rate constant (with units of reciprocal time), and *cA* is the molar concentration of species A.
- For single-phase reactions, the rate constant is typically a strong function of reaction temperature given by the Arrhenius relation,

$$k = k_0 \exp(-E/RT) \tag{2-63}$$

- where *k0* is the frequency factor,
- *E* is the activation energy,
- and *R* is the gas constant.
- The expressions in (2-62) and (2-63) are based on theoretical considerations, but model parameters *ko* and *E* are usually determined by fitting experimental data.
- Thus, these two equations can be considered to be *semi-empirical* relations.
- The inlet stream consists of pure component A with molar concentration, *cAi*·
- A cooling coil is used to maintain the reaction mixture at the desired operating temperature by removing heat that is released in the exothermic reaction.



- initial CSTR model development is based on three assumptions:
- **1.** The CSTR is perfectly mixed.
- 2. The mass densities of the feed and product streams are equal and constant. They are denoted by p.
- **3.** The liquid volume *V* in the reactor is kept constant by an overflow line.



For these assumptions, the unsteady-state mass balance for the CSTR is:

$$\frac{d(\rho V)}{dt} = \rho q_i - \rho q \qquad (2\text{-}64)$$

Because V and ρ are constant, (2-64) reduces to

$$q = q_i \qquad (2\text{-}65)$$

Thus, even though the inlet and outlet flow rates may change due to upstream or downstream conditions, Eq. 2-65 must be satisfied at all times. In Fig. 2.6, both flow rates are denoted by the symbol q.

For the stated assumptions, the unsteady-state component balances for species A (in molar units) is

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - Vkc_A \tag{2-66}$$

This balance is a special case of the general component balance in Eq. 2-7.

Next, we consider an unsteady-state energy balance for the CSTR. But first we make five additional assumptions:

4. The thermal capacitances of the coolant and the cooling coil wall are negligible compared to the thermal capacitance of the liquid in the tank.

5. All of the coolant is at a uniform temperature, T_c. (That is, the increase in coolant temperature as the coolant passes through the coil is neglected.)
6. The rate of heat transfer from the reactor contents to the coolant is given by

$$Q = UA(T_c - T) \tag{2-67}$$

   where U is the overall heat transfer coefficient and A is the heat transfer area. Both of these model parameters are assumed to be constant.
7. The enthalpy change associated with the mixing of the feed and the liquid in the tank is negligible compared with the enthalpy change for the chemical reaction. In other words, the heat of mixing is negligible compared to the heat of reaction.

8. Shaft work and heat losses to the ambient can be neglected.

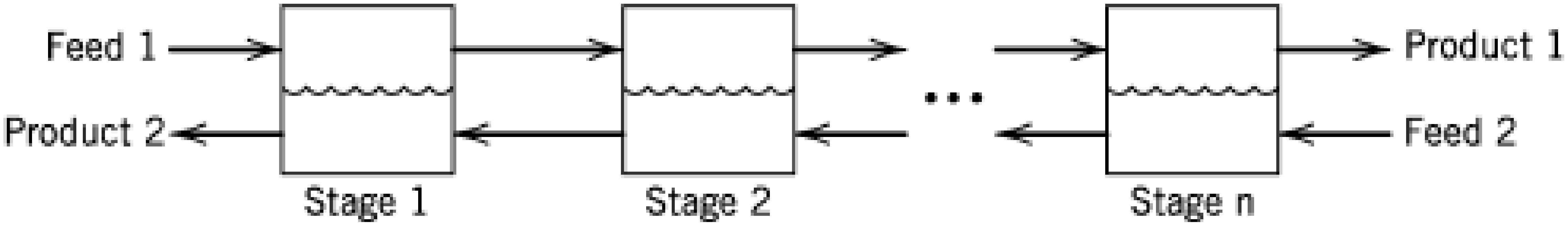
The following form of the CSTR energy balance is convenient for analysis and can be derived from Eqs. 2-62 and 2-63 and Assumptions 1–8 (Fogler, 2006; Russell and Denn, 1972),

$$V\rho C\frac{dT}{dt} = wC(T_i - T) + (-\Delta H_R)Vkc_A + UA(T_c - T) \tag{2-68}$$

where ΔH_R is the heat of reaction per mole of A that is reacted.

Staged Systems (a Three-Stage Absorber)

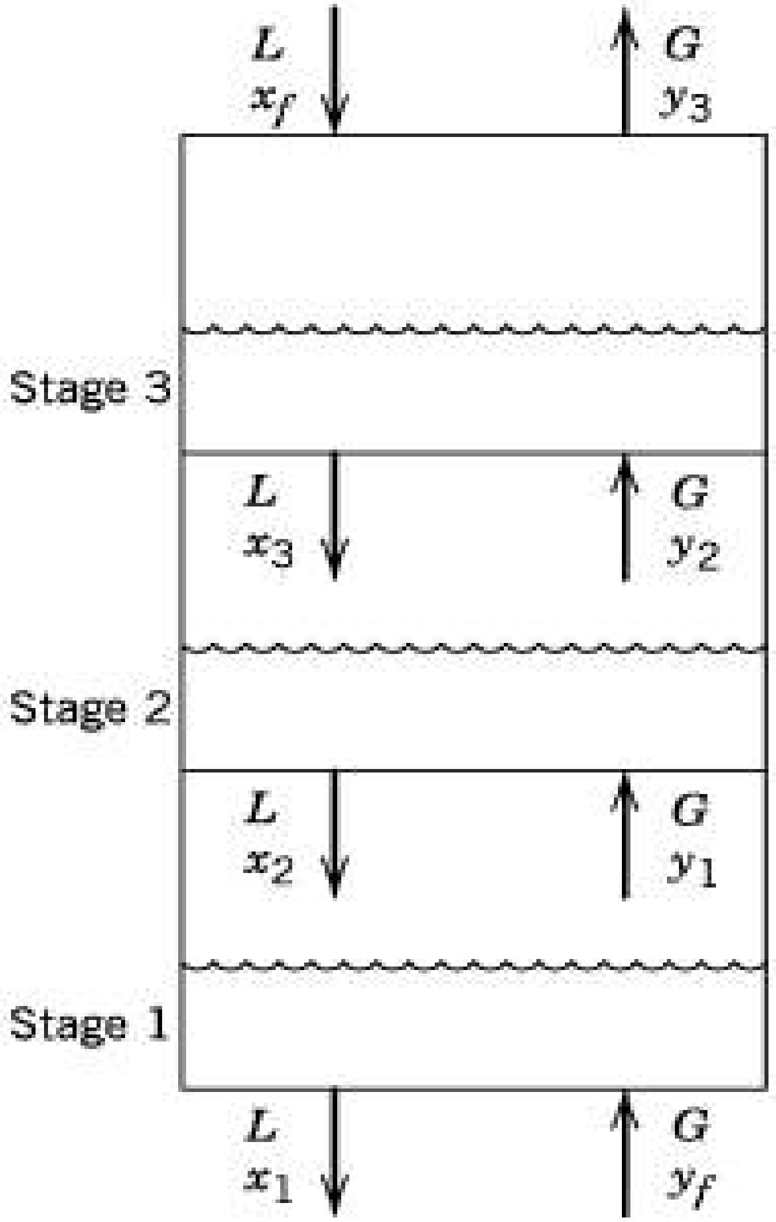
- Chemical processes, particularly separation processes, often consist of a sequence of stages. In each stage, materials are brought into intimate contact to obtain (or approach) equilibrium between the individual phases. The most important examples of staged processes include **distillation, absorption, and extraction**.
- The stages are usually arranged as a cascade with immiscible or partially miscible materials (the separate phases) flowing either cocurrently or countercurrently. Countercurrent contacting, shown in Fig. usually permits the highest degree of separation to be attained in a fixed number of stages



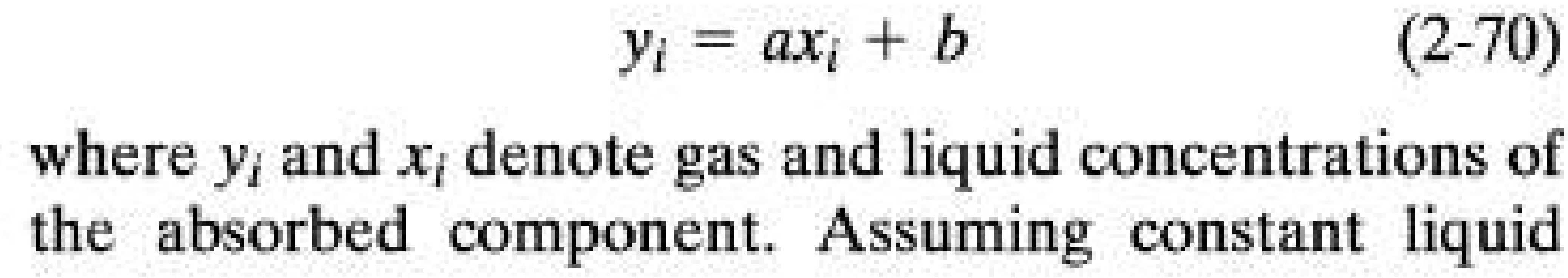
- The feeds to staged systems may be introduced at each end of the process, as in absorption units, or a single feed may be introduced at a middle stage, as is usually the case with distillation.
- The stages may be physically connected in either a vertical or horizontal configuration, depending on how the materials are transported, that is, whether pumps are used between stages, and so forth.

- we consider **a gas-liquid absorption process**, because its dynamics are somewhat simpler to develop than those of distillation and extraction processes. At the same time, it illustrates the characteristics of more complicated countercurrent staged processes.

- For the three-stage absorption unit shown in Fig. a gas phase is introduced at the bottom (molar flow rate G) and a single component is to be absorbed into a liquid phase introduced at the top (molar flow rate *L,* flowing countercurrently).
- A practical example of such a process is the removal of sulfur dioxide (S02) from combustion gas by use of a liquid absorbent. The gas passes up through the perforated (sieve) trays and contacts the liquid cascading down through them.



- series of weirs and down comers typically are used to retain a significant holdup of liquid on each stage while forcing the gas to flow upward through the perforations.
- Because of intimate mixing, we can assume that the component to be absorbed is in equilibrium between the gas and liquid streams leaving each stage *i.*
- a simple linear relation is often assumed.
- For stage *i*



$$y_i = ax_i + b \tag{2-70}$$

where y_i and x_i denote gas and liquid concentrations of the absorbed component. Assuming constant liquid holdup H and perfect mixing on each stage, and neglecting the holdup of gas, the component material balance for any stage i is

$$H\frac{dx_i}{dt} = G(y_{i-1} - y_i) + L(x_{i+1} - x_i) \qquad (2\text{-}71)$$

In Eq. 2-71 we also assume that molar liquid and gas flow rates L and G are unaffected by the absorption, because changes in concentration of the absorbed component are small, and L and G are approximately constant. Substituting Eq. 2-70 into Eq. 2-71 yields

$$H\frac{dx_i}{dt} = aGx_{i-1} - (L + aG)x_i + Lx_{i+1} \qquad (2\text{-}72)$$

Dividing by L and substituting $\tau = H/L$ (the stage liquid residence time), $\mathcal{S} = aG/L$ (the *stripping factor*), and $K = G/L$ (the gas-to-liquid ratio), the following model is obtained for the three-stage absorber:

$$\tau \frac{dx_1}{dt} = K(y_f - b) - (1 + \delta)\, x_1 + x_2 \qquad (2\text{-}73)$$

$$\tau \frac{dx_2}{dt} = \delta x_1 - (1 + \delta) x_2 + x_3 \qquad (2\text{-}74)$$

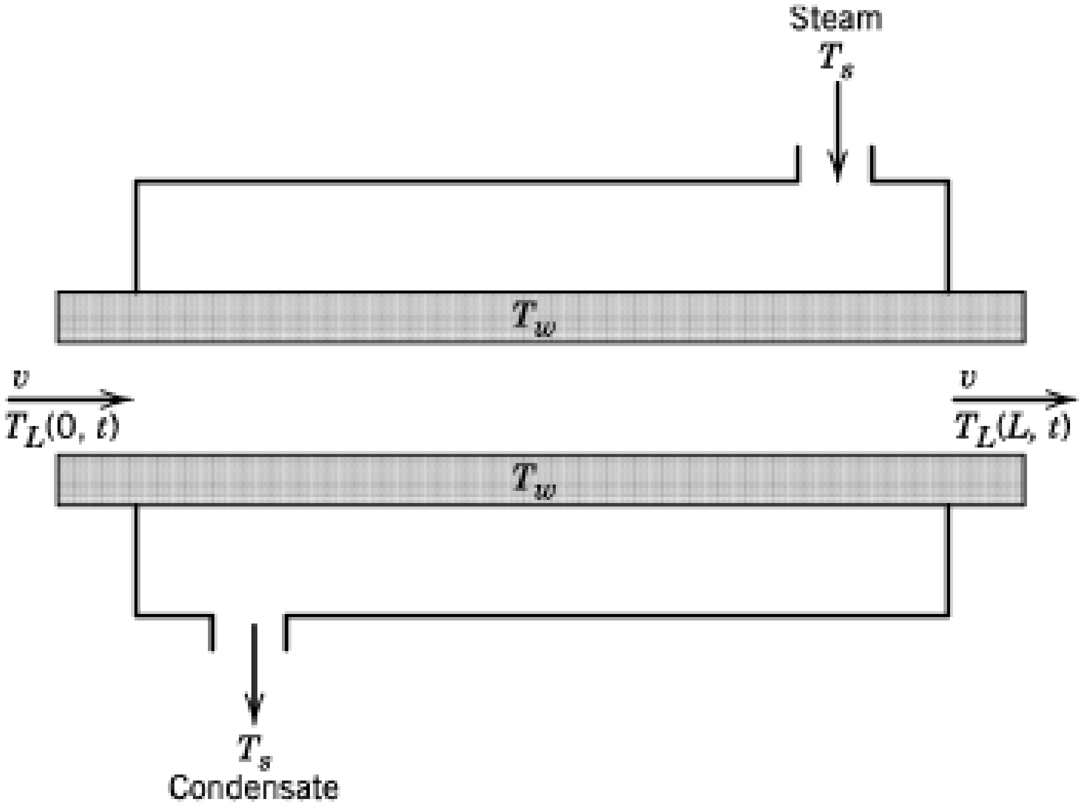
$$\tau \frac{dx_3}{dt} = \delta x_2 - (1 + \delta) x_3 + x_f \qquad (2\text{-}75)$$

In the model of (2-73) to (2-75) note that the individual equations are linear but also coupled, meaning that each output variable—x_1, x_2, x_3—appears in more than one equation. This feature can make it difficult to convert these three equations into a single higher-order equation in one of the outputs, as was done in Eq. 2-49.

Distributed Parameter Systems (the Double-Pipe Heat Exchanger)

- All of the process models discussed up to this point have been of the ***lumped parameter*** type, meaning that any dependent variable can be assumed to be a function only of time and not of spatial position.
- For the stirred tank systems discussed earlier, we assumed that any spatial variations of the temperature or concentration within the liquid could be neglected.
- Perfect mixing in each stage was also assumed for the absorber.
- Even when perfect mixing cannot be assumed, a lumped or average temperature may be taken as representative of the tank contents to simplify the process model.

- Figure shown illustrates a double-pipe heat exchanger where **a** fluid flowing through the inside tube with velocity ***v*** is heated by steam condensing in the outer tube.
- If the fluid is assumed to be in plug flow, the temperature of the liquid is expressed as *T L(Z, t)* where *z* denotes distance from the fluid inlet.
- The fluid heating process is truly distributed parameter; at any instant in time there is a temperature profile along the inside tube.
- The steam condensation, on the other hand, might justifiably be treated as a lumped process, because the steam temperature *Ts(t)* can be assumed to be a function of the condensation pressure, itself presumably a function only of time and not a function of position.
- We also assume that the wall temperature *T w(Z, t)* is different from T_L and T_S due to the thermal capacitance and resistances.



- In developing a model for this process, assume that the liquid enters at temperature *T* L(O, *t)*-that is, at *z* = 0.
- Heat transfer coefficients (steam-to-wall *hs* and wall-to liquid *hL)* can be used to approximate the energy transfer processes.
- We neglect the effects of axial energy conduction, the resistance to heat transfer within the metal wall, and the thermal capacitance of the steam condensate.
- A distributed parameter model for the heat exchanger can be derived by applying Eq. 2-8 over a differential tube length Δ*z* of the exchanger. In such a *shell* energy balance, the partial differential equation is obtained by taking the limit as *Δz* →0.
- Using the conversation law, Eq. 2-8, the following PDE results

$$\rho_L C_L S_L \frac{\partial T_L}{\partial t} = -\rho_L C_L S_L v \frac{\partial T_L}{\partial z} + h_L A_L (T_w - T_L) \qquad (2\text{-}76)$$

where the following parameters are constant: ρ_L = liquid density, C_L = liquid heat capacity, S_L = cross-sectional area for liquid flow, h_L = liquid heat transfer coefficient, and A_L = wall heat transfer area of the liquid. This equation can be rearranged to yield

$$\frac{\partial T_L}{\partial t} = -v\frac{\partial T_L}{\partial z} + \frac{1}{\tau_{HL}}(T_w - T_L) \qquad (2\text{-}77)$$

where $\tau_{HL} = \rho_L C_L S_L / h_L A_L$ has units of time and is called the characteristic time for heating of the liquid. An energy balance for the wall gives

$$\rho_w C_w S_w \frac{\partial T_w}{\partial t} = h_s A_s(T_s - T_w) - h_L A_L(T_w - T_L) \qquad (2\text{-}78)$$

where the parameters associated with the wall are denoted by subscript w and the steam-side transport parameters are denoted by subscript s. Because T_w depends on T_L, it is also a function of time and position, $T_w(z, t)$. T_s is a function only of time, as noted above. Equation 2-78 can be rearranged as

$$\frac{\partial T_w}{\partial t} = \frac{1}{\tau_{sw}}(T_s - T_w) - \frac{1}{\tau_{wL}}(T_w - T_L) \quad (2\text{-}79)$$

where

$$\tau_{sw} = \frac{\rho_w C_w S_w}{h_s A_s} \quad \text{and} \quad \tau_{wL} = \frac{\rho_w C_w S_w}{h_L A_L} \quad (2\text{-}80)$$

are characteristic times for the thermal transport processes between the steam and the wall and the wall and the liquid, respectively.

- To be able to solve Eqs. 2-77 and 2-79, boundary conditions for both T_L and T_w at time $t = 0$ are required.
- Assume that the system initially is at steady state ($\partial T_L/\partial t = \partial T_w/\partial t = 0$; $T_s(0)$ is known).
- The steady state profile, *h(z,* 0), can be obtained by integrating Eq. 2-77 with respect to *z* simultaneously with solving the steady-state version of Eq. 2-79, an algebraic expression.
- Note that the steady-state version of (2-77) is an ODE in *z,* with $T_L(0, 0)$ as the boundary condition.
- $T_w(Z, 0)$ is found algebraically from T_s and $T_L(z, 0)$.

To obtain ODE models with time as the independent variable, the z dependence is eliminated by discretization. In Fig. 2.12 the double-pipe heat exchanger has been redrawn with a set of grid lines to indicate points at which the liquid and wall temperatures will be evaluated. We now rewrite Eqs. 2-77 and 2-79 in terms of the liquid and wall temperatures $T_L(0), T_L(1), \ldots, T_L(N)$ and $T_w(0), T_w(1), \ldots, T_w(N)$. Utilizing the backward difference approximation for the derivative $\partial T_L/\partial z$ yields

$$\frac{\partial T_L}{\partial z} \approx \frac{T_L(j) - T_L(j-1)}{\Delta z} \tag{2-81}$$

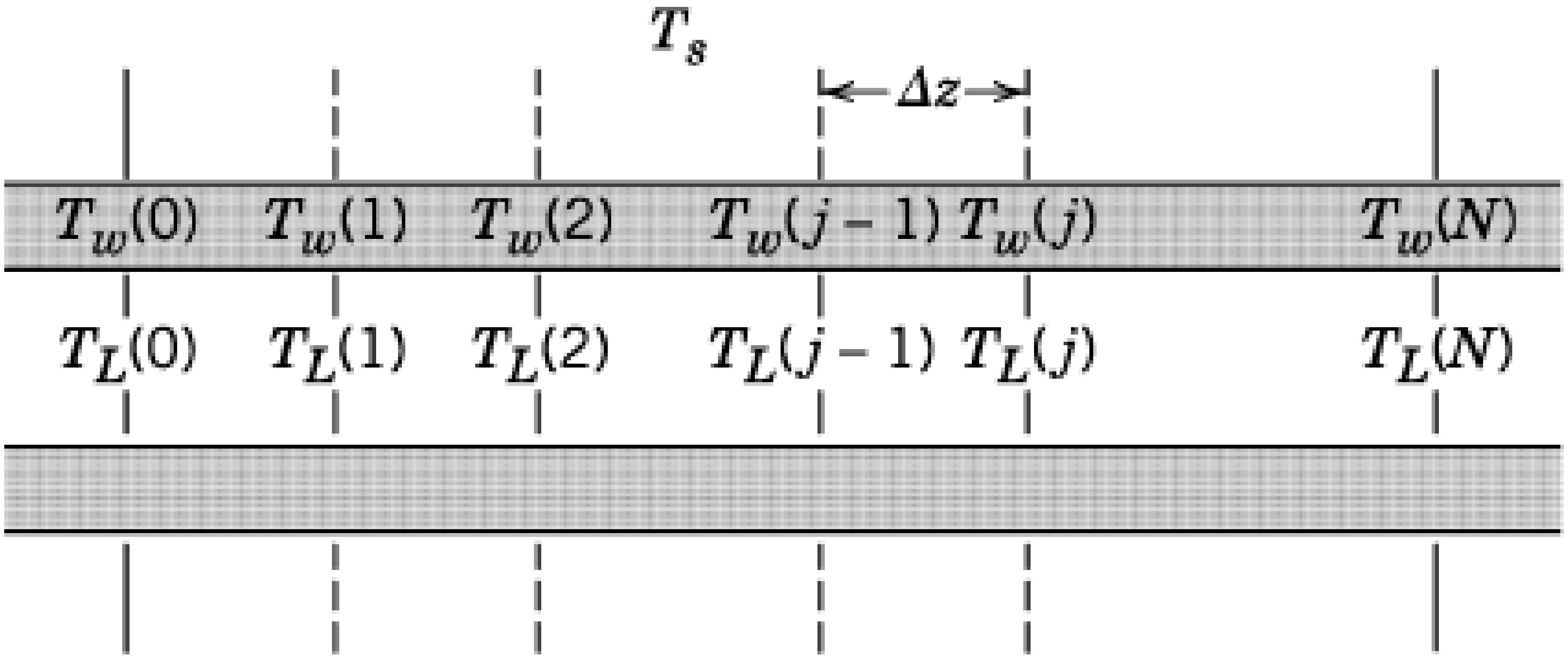


Figure 2.12 Finite-difference approximations for double-pipe heat exchanger.

where $T_L(j)$ is the liquid temperature at the jth node (discretization point). Substituting Eq. 2-81 into Eq. 2-77, the equation for the jth node is

$$\frac{dT_L(j)}{dt} = -v\frac{T_L(j) - T_L(j-1)}{\Delta z} + \frac{1}{\tau_{HL}}[T_w(j) - T_L(j)]\ (j = 1, \ldots, N) \tag{2-82}$$

The boundary condition at $z = 0$ becomes

$$T_L(0, t) = T_F(t) \tag{2-83}$$

where $T_F(t)$ is a specified forcing (input) function. Rearranging Eq. 2-82 yields

$$\frac{dT_L(j)}{dt} = \frac{v}{\Delta z} T_L(j-1) - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_L(j) + \frac{1}{\tau_{HL}} T_w(j) \quad (j = 1, \ldots, N) \tag{2-84}$$

Similarly, for the wall equation,

$$\frac{dT_w(j)}{dt} = -\left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_w(j) + \frac{1}{\tau_{wL}} T_L(j) + \frac{1}{\tau_{sw}} T_s(j) \quad (j = 1, \ldots, N) \tag{2-85}$$

$$\frac{dT_{L1}}{dt} = \frac{v}{\Delta z} T_F(t) - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_{L1} + \frac{1}{\tau_{HL}} T_{w1} \tag{2-86}$$

$$\frac{dT_{L2}}{dt} = \frac{v}{\Delta z} T_{L1} - \left(\frac{v}{\Delta z} + \frac{1}{\tau_{HL}} \right) T_{L2} + \frac{1}{\tau_{HL}} T_{w2} \tag{2-87}$$

$$\frac{dT_{w1}}{dt} = -\left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_{w1} + \frac{1}{\tau_{wL}} T_{L1} + \frac{1}{\tau_{sw}} T_s(t) \tag{2-88}$$

$$\frac{dT_{w2}}{dt} = -\left(\frac{1}{\tau_{sw}} + \frac{1}{\tau_{wL}} \right) T_{w2} + \frac{1}{\tau_{wL}} T_{L2} + \frac{1}{\tau_{sw}} T_s(t) \tag{2-89}$$

where the node number has been denoted by the second subscript on the output variables to simplify the notation. Equations 2-86 to 2-89 are coupled, linear, ordinary differential equations.