Fundamentals of Communication systems

Third Year-Control Eng.   Theoretical : 2 hrs./week (Semester)

e Practical : 2

1-Introduction to Communication and System
- -Basic Elements of a Communication System.
- -Mathematical Background.
- -Fourier Series Transformation.
- -power and energy.

2-Amplitude Modulation
- -Normal AM.
- -Single Side Band.
- -Double Side Band Suppressed Carrier.
- -Super Heterodyne Transmitter and Receiver.

3-Angular Modulation
- -Frequency Modulation.
- -Phase Modulation.

4-Pulse Modulation
- -Pulse Amplitude Modulation.
- - Pulse Width Modulation.
- - Pulse Position Modulation.
- - Pulse Code Modulation.
- -Sampling Theory.

5-Transmission Line Concepts and S/N Ratio.

References:

1- Introduction to communication systems , F. G. Stremter

2- Modern Digital and Analog communication systems , B. P. Lathi

3- Communication systems , A. Bruce Carlson

4- Signals and systems with MATLAB Applications , Orchard Puplications

Mathematical Relations

Trigonometric identities

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$e^{j2\alpha} + e^{j2\beta} = 2\cos(\alpha - \beta)e^{j(\alpha+\beta)}$$

$$e^{j2\alpha} - e^{j2\beta} = j2\sin(\alpha - \beta)e^{j(\alpha+\beta)}$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}) = \sin(\theta + 90^\circ)$$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) = \cos(\theta - 90^\circ)$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta)$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha - \beta) - \frac{1}{2}\cos(\alpha + \beta)$$

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha - \beta) + \frac{1}{2}\sin(\alpha + \beta)$$

$$A\cos(\theta + \alpha) + B\cos(\theta + \beta) = C\cos\theta - S\sin\theta = R\cos(\theta + \phi)$$

where :

$$C = A\cos\alpha + B\cos\beta$$

$$S = A\sin\alpha + B\sin\beta$$

$$R = \sqrt{C^2 + S^2} = \sqrt{A^2 + B^2 + 2AB\cos(\alpha - \beta)}$$

$$\phi = \arctan\frac{S}{C} = \arctan\frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}$$

Definite Integral

$$\int \sin axdx = \frac{-1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int e^{ax} dx = \frac{1}{a} e_{ax}$$

$$\int xe^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax} \sin bxdx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bxdx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx)$$

Elements of Communication System

Communication system: are found wherever information is to be transmitter from one point, call the transmitter (source), to anther point , called the receiver (destination), TV, radio,…..

We can identify two distinct message (information)(electrical signal) categories analog and digital.

An analog message: is a physical quantity that varies with time. (acoustical quantity, light intensity in TV).

A digital message : is an ordered sequence of symbols selected from a finite set of discrete elements (a listing of hourly temperature reading, the keys at a computer).

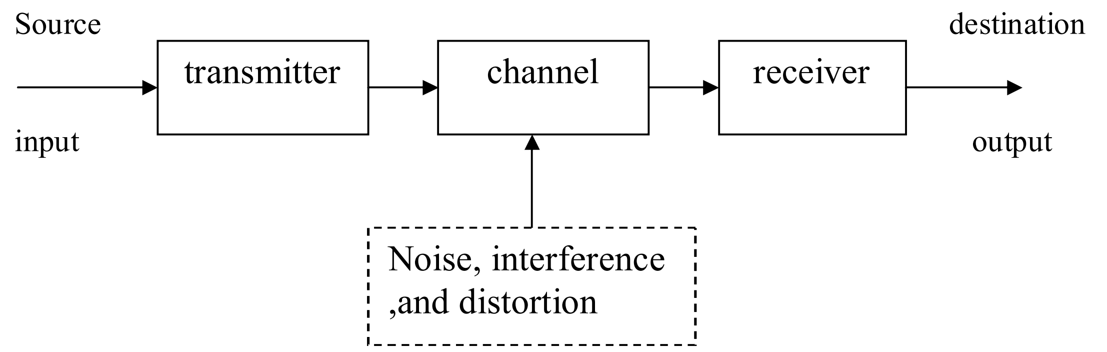
Most communication system have:

1. input
2. output transducer
3. channel

the input converts the message to an electrical signal, say a voltage or current, and anther transducer at the destination converts the output signal to the desired message form.

Types of communication systems:

1. Simplex Transmission(SX) :represents way of simplex transmission.
2. A full-duplex(FDX) system has a channel that allows simultaneous transmission in both directions.
3. A alf-duplex (HDX) system allows transmission in either direction but not at the same time.



Element of communication system

Transducer: the input massage is usually not electrical, hence an input transducer is required to convert the message to a signal (voltage or current). Similarly anther transducer at the destination converts the output signal to the appropriate message form.

Channel: it is the transmission medium which provides the electrical connection between the source and the receiver ex. Wire, coaxial cable, optical fiber, ionosphere, free space, ……

The channel (regardless of the type) degrades the transmitted signal in a number of ways :-

1-Attenuation: it is defined as a reduction of signal strength. (increase with channel length).

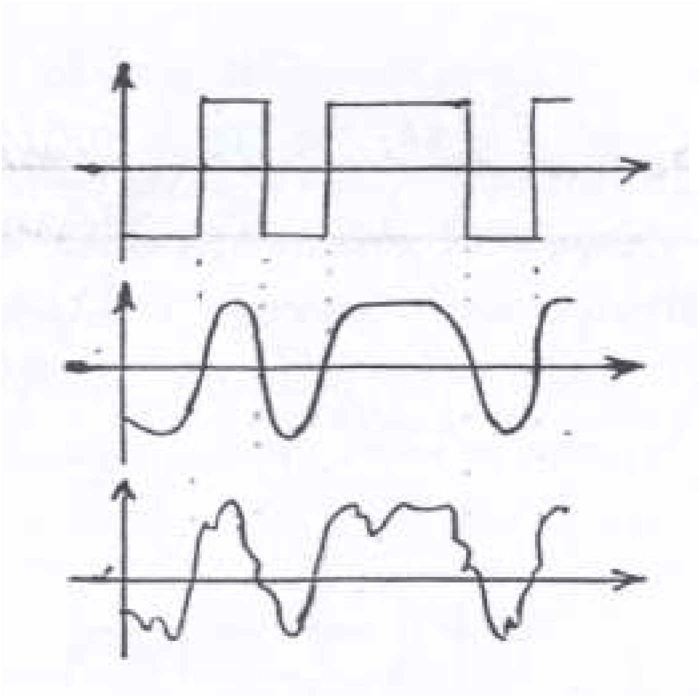
2-Distortion: is waveform perturbation caused by imperfect response of the system to the desired signal itself. Distortion may be corrected, or at least reduced with the help of special filters called equalizers.

3-Interference: is contamination by extraneous signals from human source, other transmitters, machinery, switching circuits, ……,

Filtering removes interference.

4-Noise: it is a random and undesirable signal from external and internal causes.It is defined as the ratio of the signal power to the noise power:

Signal to Noise Ratio $S/N = 10 \log \left(\frac{P_s}{P_N} \right)$, (80 dB)the beast.



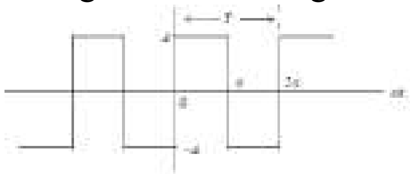
Transmitted signal

Received distorted signal

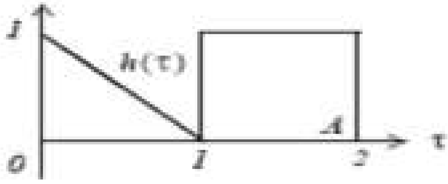
Received distorted signal with noise

Classification of signal :

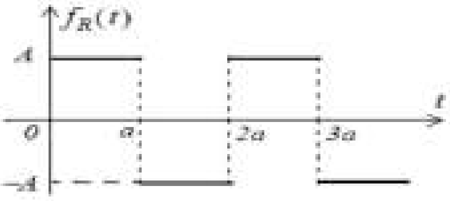
1- periodic signal : it is the signal which repeats itself after a fixed length of time .



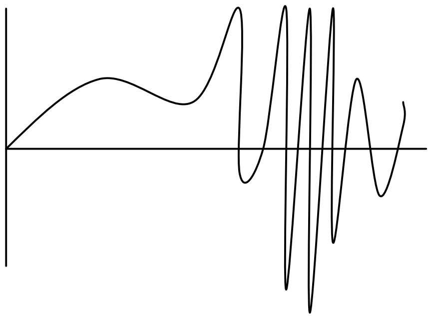
2- non periodic signal : it is the signal which not repeats itself after a fixed length of time .



3- Determinstic signal : it is the signal which can be mathematical expression.



4- Random signal : it is signal which there is uncertainty in its values.



5- Energy signal : the signal usually exists for only a finite interval of time and have finite energy $\int_{-\infty}^{\infty} \left| f(t) \right|^2 dt < \infty$

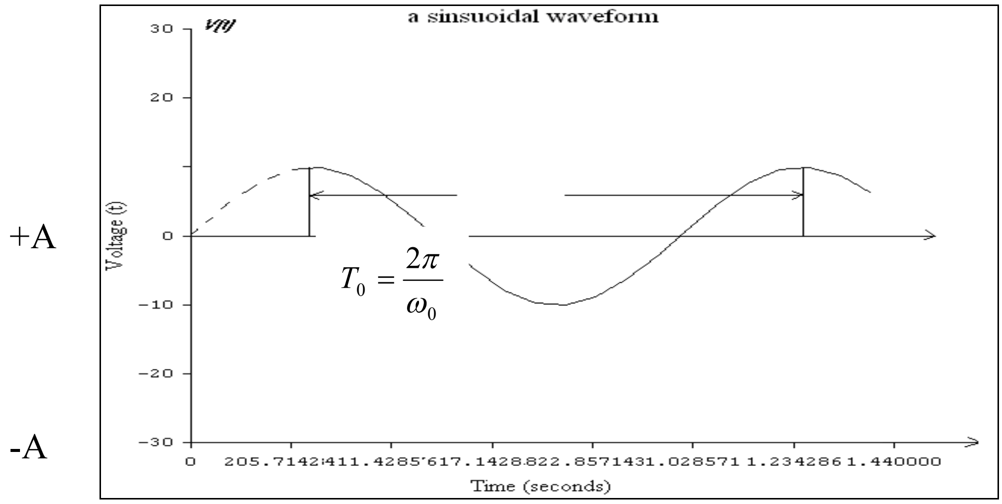
6- Power signal : the signal has finite and non-zero average power.

Signal and spectra:

A signal is time-varying quantities such as voltage or current in the time domain.

Time domain ⟶ Fourier series ⟶ frequency domain
OR transform

The freq.-domain description is called the spectrum. as the first step in spectral analysis we must write equation representing signals as functions of time.



$$V(t) = A\cos(\omega_0 t + \phi)$$

A : peak value or amplitude.

ω_0 : radian frequency.

ϕ : phase angle.

T_0 : period = $\frac{2\pi}{\omega_0}$ (cyclical frequency fo) (sycl/sec)or(hertz).

$$f_0 = \frac{1}{T_0} = \frac{\omega_0}{2\pi}$$

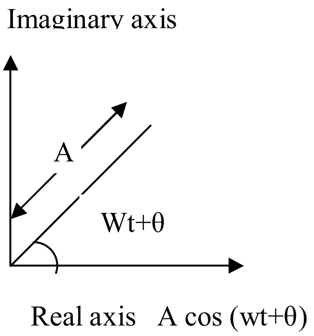
Vector in complex plane:

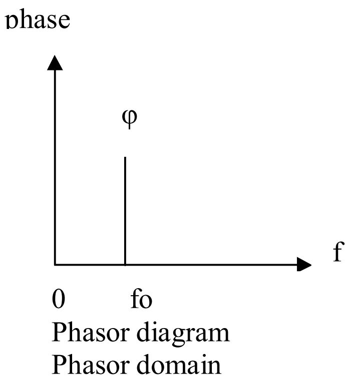
$$\boxed{e^{\pm j\theta} = \cos\theta \pm j\sin\theta} \quad ,$$

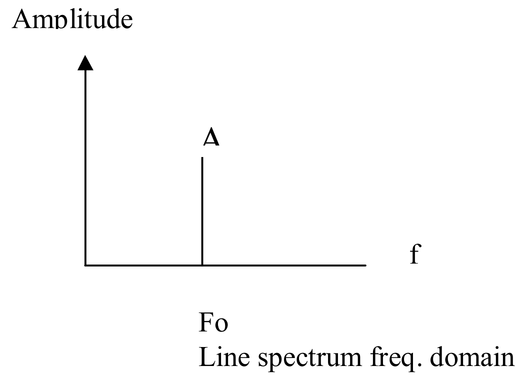
Let $\theta = \omega_o + \phi$, $j = \sqrt{-1}$

$$A\cos(w_o t + \phi) = A\,\mathrm{Re}[e^{j(w_o t+\phi)}] = \mathrm{Re}[Ae^{j\phi}\cdot e^{jw_o t}]$$

Re : real part. A phaser representation







Not :

1) phase angles will be measured with respect to cosine waves (positive real axis), hence sine wave need to be converted to cosine.

$$\sin wt = \cos(wt - 90^0)$$

2) Amplitude as always being a positive quantity.

Fourier series (in rectangular form)

periodic waveform is a waveform that repeats itself after some time, any periodic waveform f(t) can be expressed as :

$$f(t) = \frac{1}{2}a_0 + a_1\cos\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + a_4\cos 4\omega t + \dots$$
$$+ b_1\sin\omega t + b_2\sin 2\omega t + b_3\sin 3\omega t + b_4\sin 4\omega t + \dots$$

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n\sin n\omega t)$$

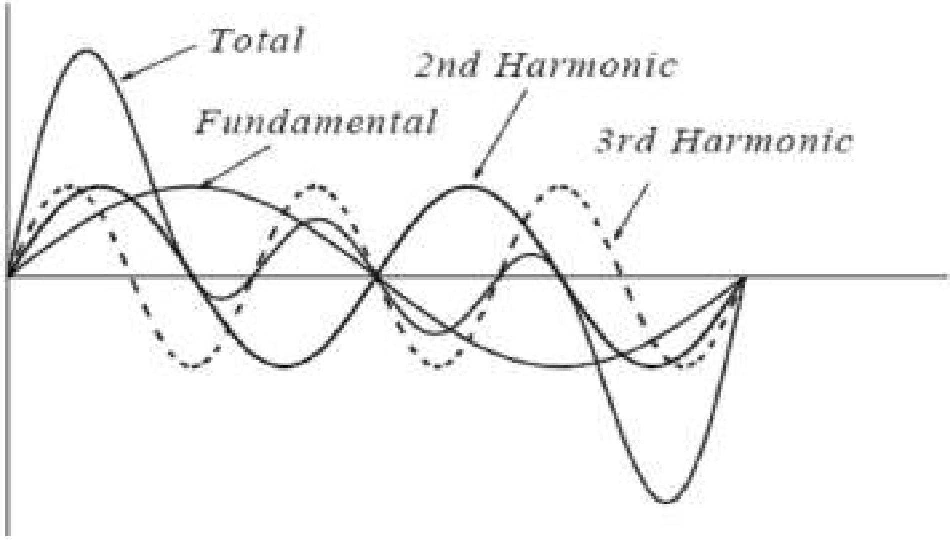
Where :

ao/2 : is a constant , and represents the DC (average) component of f(t) .

f(*t*) *:* represents some voltage v(t) , or current i(t).

a1 and b1: represent the fundamental frequency component w.

a2 and b2 : represent the second harmonic component 2w.



Summation of a fundamental, second and third harmonic

The coefficients *a0 , an , bn* and are found from the following relations :

$$\frac{1}{2}a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)dt$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\,dt$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\,dt$$

If w=1 , m, n are integer then

$$\int_0^{2\pi} \sin mt\,dt = 0$$

$$\int_0^{2\pi} \cos mt\,dt = 0$$

$$\int_0^{2\pi} (\sin mt)(\cos nt)\,dt = 0$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\int_0^{2\pi} (\sin mt)(\sin nt)\,dt = 0$$

$$(\sin x)(\sin y) = \frac{1}{2}[\cos(x-y) - \cos(x-y)]$$

$$\int_0^{2\pi} (\cos mt)(\cos nt)\,dt = 0$$

$$(\cos x)(\cos y) = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

$$\int_0^{2\pi} (\sin mt)^2\,dt = \pi$$

$$\int_0^{2\pi} (\cos mt)^2\,dt = \pi$$

Symmetry

We will discuss three types of symmetry that can be used to facilitate the computation of the trigonometric Fourier series form. These are:

1. *Odd symmetry* − If a waveform has odd symmetry, that is, if it is an odd function, the series will consist of sine terms only. In other words, if f(t) is an odd function, all the *ai* coefficients including , will be zero.

2- *Even symmetry* − If a waveform has even symmetry, that is, if it is an even function, the series will consist of cosine terms only, and *a0* may or may not be zero. In other words, if f(t) is an even function, all the *bi* coefficients will be zero.

3. *Half-wave symmetry* − If a waveform has half-wave symmetry (to be defined shortly), only odd (odd cosine and odd sine) harmonics will be present. In other words, all even (even cosine and even sine) harmonics will be zero.

We recall that odd functions are those for which $-f(-t) = f(t)$

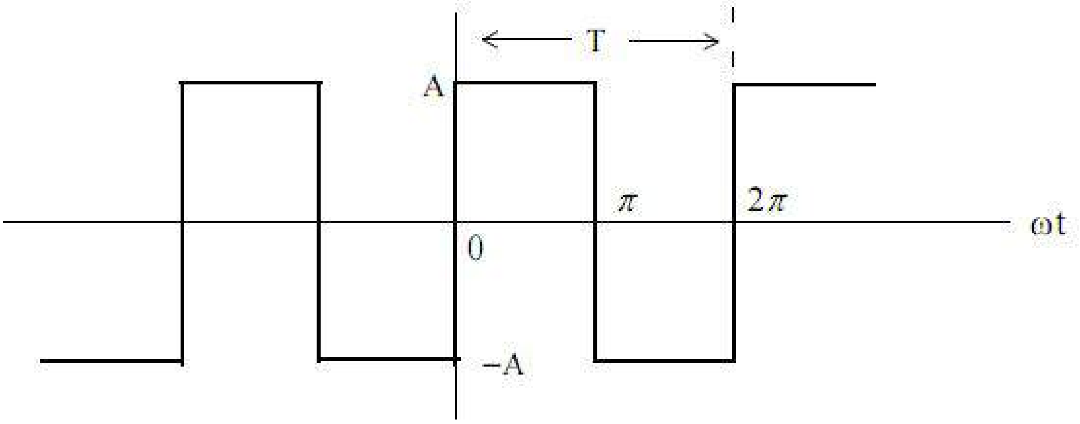
and even functions are those for which $f(-t) = f(t)$

half-wave symmetry, any periodic function with period , is expressed as $f(t) = f(t + T)$. that is, the function with value f(t) at any time , will have the same value again at a later time $t + T$.

A periodic waveform with period , has half-wave symmetry if $-f(t+T/2) = f(t)$. that is, the shape of the negative half-cycle of the waveform is the same as that of the positive half cycle, but inverted.

<u>Example :</u>

Compute the trigonometric Fourier series of the square waveform of Fig ure , by integrating from0to Л , and multiplying the result by 4 .



Square waveform

<u>Solution:</u>

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\,dt = \frac{1}{\pi}\left[\int_0^{\pi} A\cos nt\,dt + \int_{\pi}^{2\pi}(-A)\cos nt\,dt\right] = \frac{A}{n\pi}\left(\sin nt\big|_0^{\pi} - \sin nt\big|_{\pi}^{2\pi}\right)$$

$$= \frac{A}{n\pi}(\sin n\pi - 0 - \sin n2\pi + \sin n\pi) = \frac{A}{n\pi}(2\sin n\pi - \sin n2\pi)$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} A\,dt + \int_{\pi}^{2\pi}(-A)\,dt\right] = \frac{A}{\pi}(\pi - 0 - 2\pi + \pi) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\,dt = \frac{1}{\pi}\left[\int_0^{\pi} A\sin nt\,dt + \int_{\pi}^{2\pi}(-A)\sin nt\,dt\right] = \frac{A}{n\pi}\left(-\cos nt\Big|_0^{\pi} + \cos nt\Big|_{\pi}^{2\pi}\right)$$

$$= \frac{A}{n\pi}(-\cos n\pi + 1 + \cos 2n\pi - \cos n\pi) = \frac{A}{n\pi}(1 - 2\cos n\pi + \cos 2n\pi)$$

For n = even

$$b_n = \frac{A}{n\pi}(1-2+1) = 0$$

For n= odd , n=2K+1 , K :constant

K=0 ,n=1

K=1 ,n=3

K=2 ,n=5

K=3 ,n=7

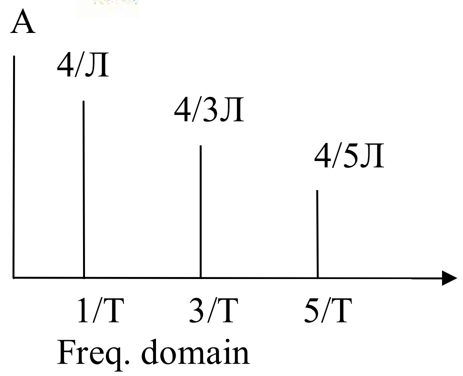
K=4 ,n=9

$$b_n = \frac{A}{n\pi}(1+2+1) = \frac{4A}{n\pi}$$

$$b_1 = \frac{4A}{\pi}$$

$$b_3 = \frac{4A}{3\pi}$$

$$b_5 = \frac{4A}{5\pi}$$

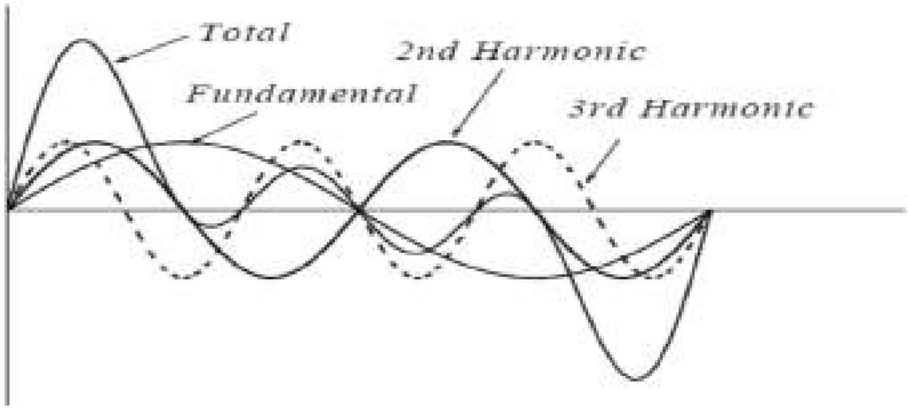


$$f(t) = \frac{4A}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right) = \frac{4A}{\pi}\sum_{n=odd}\frac{1}{n}\sin n\omega t$$

n=1 , f(t)=(4/Л) sin w t

n=3 , f(t)=(4/3Л) sin3wt

n=K , f(t)=(4/KЛ) sin K w t



$$b_n = 4\frac{1}{\pi}\int_0^{\pi/2} f(t)\sin nt\,dt = \frac{4A}{n\pi}\left(-\cos nt\Big|_0^{\pi/2}\right) = \frac{4A}{n\pi}\left(-\cos n\frac{\pi}{2} + 1\right)$$

For n= odd,

$$b_n = \frac{4A}{n\pi}(-0+1) = \frac{4A}{n\pi}$$

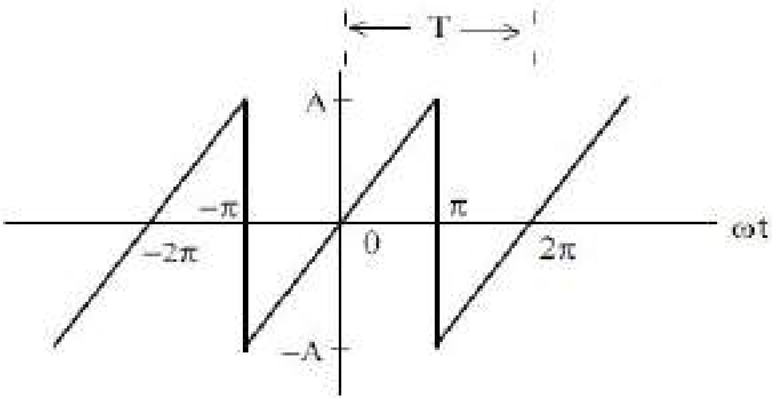
In phase domain

$$\phi = \tan^{-1}\frac{b_n}{a_n}$$

Phase domain

<u>Example :</u>

Compute the trigonometric Fourier series of the sawtooth wavef orm of Figure



Sawtooth waveform

<u>Solution:</u>

This waveform is an odd function the DC component is zero

$$f(t) = \begin{cases} \frac{A}{\pi}t & 0 < t < \pi \\ \frac{A}{\pi}t - 2A & \pi < t < 2\pi \end{cases}$$

we can choose the limits from 0 to +Л ,

$$b_n = \frac{2}{\pi}\int_0^{\pi} \frac{A}{\pi} t\sin nt dt = \frac{2A}{\pi^2}\int_0^{\pi} t\sin nt dt = \frac{2A}{\pi^2}\left(\frac{1}{n^2}\sin nt - \frac{t}{n}\cos nt\right)\Big|_0^{\pi}$$

$$= \frac{2A}{n^2\pi^2}(\sin nt - nt\cos nt)\Big|_0^{\pi} = \frac{2A}{n^2\pi^2}(\sin n\pi - n\pi\cos n\pi)$$

$$\int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$

We observe that:

1- if n = even, sin nЛ =0, cos nЛ =1

$$b_n = \frac{2A}{n^2\pi^2}(-n\pi) = -\frac{2A}{n\pi}$$

the even harmonics have negative coefficients.

2- if n = odd, sin nЛ = 0, cosnЛ =1

$$b_n = \frac{2A}{n^2\pi^2}(n\pi) = \frac{2A}{n\pi}$$

$$f(t) = \frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \ldots\right) = \frac{2A}{\pi}\sum(-1)^{n-1}\frac{1}{n}\sin n\omega t$$

The Exponential Form of the Fourier Series

$$F(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnwt}$$

N = 0, 1, 2, ……..

Cn : series coefficients (complex quantities)

$$\boxed{C_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}\,d(\omega t)}$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

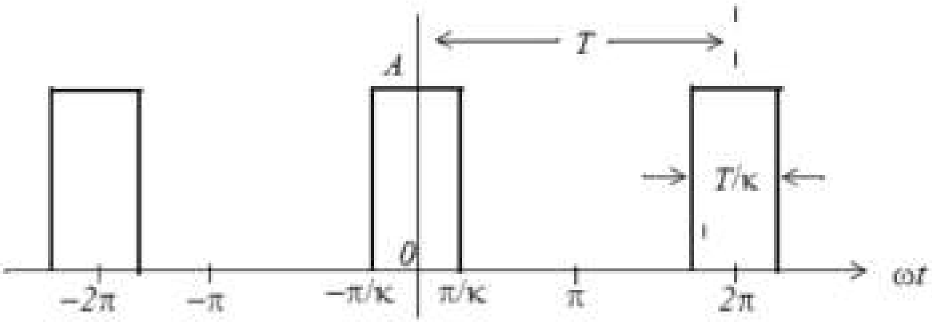
$$\sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{j2}$$

$$C_n = \frac{a_n - jb_n}{2} \qquad C_{-n} = \frac{a_n + jb_n}{2}$$

Co :the DC component equals the average value of the signal.

Example :

Compute the exponential Fourier series for the rectangular pulse train, and plot its line spectra. Assume w=1



Solution:

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} Ae^{-jnt}dt = \frac{A}{2\pi}\int_{-\pi/k}^{\pi/k} e^{-jnt}dt$$

The value of the average (DC component) is found by letting n=0 is found by letting

$$C_0 = \frac{A}{2\pi}t\Big|_{-\pi/k}^{\pi/k} = \frac{A}{2\pi}\left(\frac{\pi}{k}+\frac{\pi}{k}\right)$$

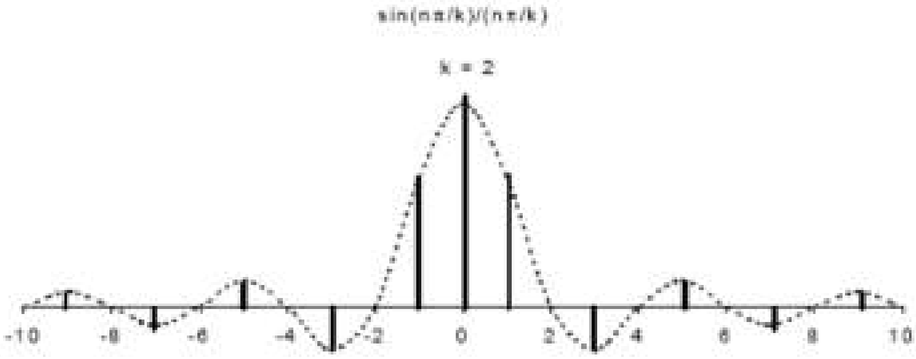
$$C_0 = \frac{A}{k}$$

$$C_n = \frac{A}{-jn2\pi} e^{-jnt} \Big|_{-\pi/k}^{\pi/k} = \frac{A}{n\pi} \cdot \frac{e^{jn\pi/k} - e^{-jn\pi/k}}{j2} = \frac{A}{n\pi} \cdot \sin\left(\frac{n\pi}{k}\right) = A \frac{\sin\left(\frac{n\pi}{k}\right)}{n\pi}$$

$$C_n = \frac{A}{k} \cdot \frac{\sin(n\pi/k)}{n\pi/k}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{A}{k} \cdot \frac{\sin(n\pi/k)}{n\pi/k}}$$

For k=2 k=5 and k=10



Line spectrum

The Fourier Transform

(For non periodic functions). Time limited is the essential condition for spectral analysis using the Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

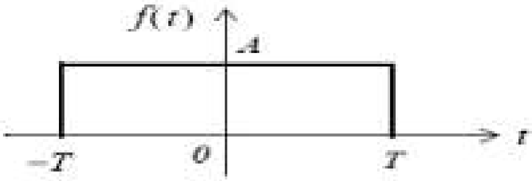
If F(w) was known, then f(t) can be found using The *Inverse Fourier transform,* is defined as

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

IFT

Example :

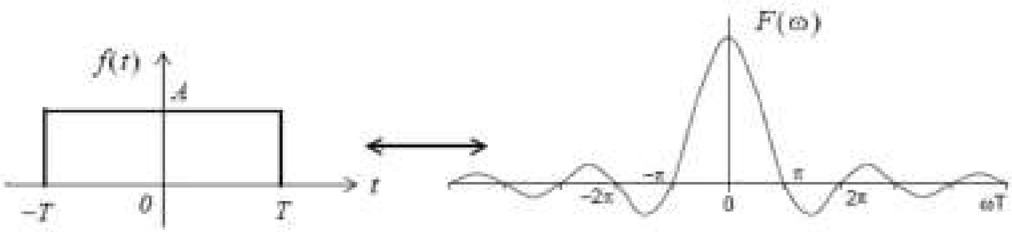
Derive the Fourier transform of the pulse



Solution:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-T}^{T} Ae^{-j\omega t}dt = \left.\frac{Ae^{-j\omega t}}{-j\omega}\right|_{-T}^{T}$$

$$= \left.\frac{Ae^{-j\omega t}}{j\omega}\right|_{T}^{-T} = \frac{A(e^{j\omega t} - e^{-j\omega t})}{j\omega} = 2A\frac{\sin\omega T}{\omega} = 2AT\frac{\sin\omega T}{\omega T}$$



Properties and Theorems of the Fourier Transform

1. Linearity

If $F1(\omega)$ is the Fourier transform of $f1(t)$, $F2(\omega)$ is the transform of $f2(t)$, and so on, the *linearity property of the Fourier transform* states that

$a1\,f1(t)+a2\,f2(t)+\ldots + an\,fn(t) \Leftrightarrow a1F1(\omega)+a2F2(\omega)+\ldots + anFn(\omega)$

Where ai is some arbitrary real constant.

2. Symmetry

If $F(\omega)$ is the Fourier transform of $f(t)$, the *symmetry property of* the Fourier transform states that

$F(t) \Leftrightarrow 2\pi f(-\omega)$, that is , if in F(t) , we replace w with t.

3. Time Scaling

If a is a real constant, and $F(\omega)$ is the Fourier transform of $f(t)$, then,

$$f(at) \Leftrightarrow \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$$

that is, the *time scaling property of the Fourier transform* states that if we replace the variable t in the time domain by *at,* we must replace the variable w in the frequency domain by ω/a , and divide $F(\omega/a)$ by the absolute value of a .

4. Time Shifting

If F(w) is the Fourier transform of f(t), then,

$$f(t-t_0) \Leftrightarrow F(\omega)e^{-j\omega t_0}$$

that is, the time shifting property of the Fourier transform states that if we shift the time function f(t) by a constant *t0* , the Fourier transform magnitude does not change, but the term ω*t0* is added to its phase angle.

5. Frequency Shifting

If F(w) is the Fourier transform of f(t) , then,

$$e^{j\omega_0 t}f(t) \Leftrightarrow F(\omega-\omega_0)$$

that is, multiplication of the time function f(t) $e^{jw_o t}$ by , where wo is a constant, results in shifting the Fourier transform by wo

6. Time Differentiation.

$$\frac{d^n}{dt^n}f(t) \Leftrightarrow (j\omega)^n F(\omega)$$

that is, the Fourier transform of $\frac{d^n}{dt^n}f(t)$, if it exists, is $(jw)^n F(w)$

7. Frequency Differentiation

$$(-jt)^n f(t) \Leftrightarrow \frac{d^n}{d\omega^n}F(\omega)$$

8. Time Integration

$$\int_{-\infty}^{t} f(\tau)d\tau \Leftrightarrow \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

9. Conjugate Time and Frequency Functions

If F(t) is the Fourier transform of the complex function f(t) , then,

$$f^*(t) \Leftrightarrow F^*(-\omega)$$

that is, if the Fourier transform of $f(t) = f_{Re}(t) + f_{Im}(t)$ is $F(\omega)$, then, the Fourier transform of $f^*(t) = f_{Re}(t) - f_{Im}(t)$ is $F^*(-\omega)$.

10. Time Convolution

If F1(t) is the Fourier transform of f1(t) , and F2(t) is the Fourier transform of f2(t) , then,

$$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega)F_2(\omega)$$

that is, convolution in the time domain, corresponds to multiplication in the frequency domain.

11. Frequency Convolution

If F1(t) is the Fourier transform of f1(t) , and F2(t) is the Fourier transform of f2(t) , then,

$$f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}F_1(\omega)*F_2(\omega)$$

that is, multiplication in the time domain, corresponds to convolution in the frequency domain divided by the constant 1/2□.

12. Area under f(t)

$$F(0) = \int_{-\infty}^{\infty} f(t)dt$$

that is, the area under a time function f(t) is equal to the value of its Fourier transform evaluated at w=0.

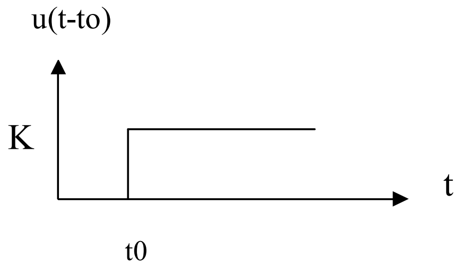
13. Area under F(w)

$$f(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)d\omega$$

that is, the value of the time function f(t) , evaluated at t=0 , is equal to the area under its Fourier F(w) transform times 1/2□ .

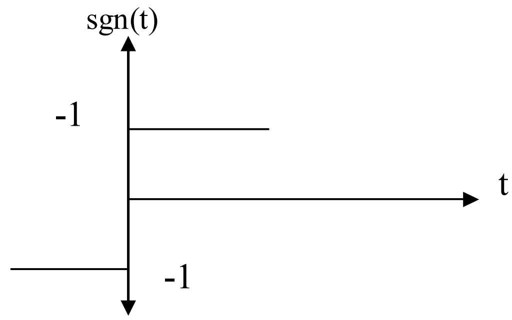
step function:

$$u(t) = \begin{cases} K \ldots\ldots\ldots\ldots t \geq 0 \\ 0 \ldots\ldots\ldots\ldots t < 0 \end{cases}$$



$$u(t - t_0) = \begin{cases} K \ldots\ldots\ldots\ldots t_0 \geq 0 \\ 0 \ldots\ldots\ldots\ldots else \end{cases}$$

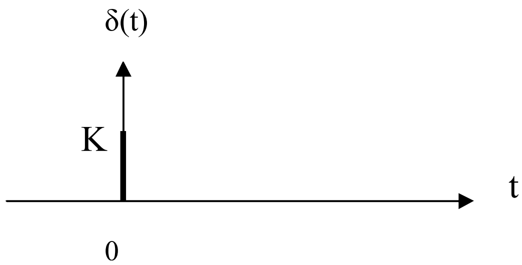
u(t) = ½ (1 + sgn (t))



$$\operatorname{sgn}(t) = \begin{cases} 1 \ldots\ldots\ldots\ldots t > 0 \\ -1 \ldots\ldots\ldots\ldots t < 0 \end{cases}$$

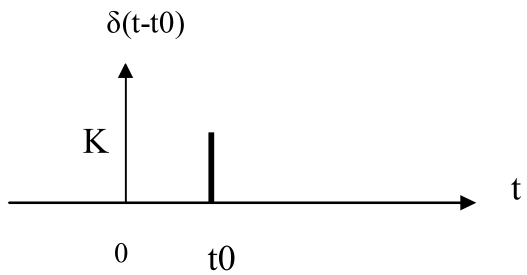
$$\int_{-\infty}^{\infty} u(t)x(t) = \int_{0}^{\infty} x(t)dt$$

Impulse functions :



$$\delta(t) = \begin{cases} K \ldots\ldots\ldots\ldots t = 0 \\ 0 \ldots\ldots\ldots\ldots else \end{cases}$$

$$\delta(t - t_0) = \begin{cases} K \ldots\ldots\ldots\ldots t = t_0 \\ 0 \ldots\ldots\ldots\ldots else \end{cases}$$



$$\int_{-\infty}^{\infty} \delta(t)dt = 1 = u(t)$$

$$\int_{-\infty}^{\infty} \delta(t - t_0)x(t) = x(t - t_0)$$

Lists of several useful F.T. pairs :

f(t)	F(w)
$\|t\|$	$\frac{-2}{w^2}$
$\delta(t)$	1
$\delta(t\text{-}to)$	e^{-jwt_0}
$u(t)$	$\pi\,\delta(w) + 1/jw$
$e^{-at}u(t)$	$1/(a+jw)$
$e^{-t^2/2a^2}$	$a\sqrt{2\pi}e^{-a^2w^2/2}$
$te^{-at}u(t)$	$\frac{1}{(a+jw)^2}$
1	$2\pi\,\delta(w)$ or $\delta(f)$
e^{jw_ot}	$2\pi\,\delta(w\text{-}wo)$

Example :

Compute the Fourier transform of $f(t) = e^{-a|t|}$ using the Fourier transform definition .

Solution: F(t)=

$$= \int_{-\infty}^{0} e^{at}e^{-j\omega t}dt + \int_{0}^{\infty} e^{-at}e^{-j\omega t}dt = \int_{-\infty}^{0} e^{(a-j\omega)t}dt + \int_{0}^{\infty} e^{-(a+j\omega)t}dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2}{\omega^2 + a^2}$$

Average power :

$$v(t) = \frac{1}{T_0}\int_{T_o} v(t)\,dt$$

$$P = |v(t)|^2 = \frac{1}{T_o}\int_{T_o} |v(t)|^2\,dt$$

$$|v(t)|^2 \cong v^2(t) \ldots\ldots\ldots \quad \ldots\ldots\ldots \quad 0 < P < \infty$$

P : a periodic power signal

V(t) = A cos (wot + φ)

Average value over all time <v(t)>=0

Then $P = \frac{A^2}{2}$

Parseval's Theorem

If F(w) is the Fourier transform of f(t), Parseval's theorem states that

$$\boxed{\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega}$$

$$P = |v(t)|^2 = \frac{1}{T_o}\int_{T_o} |v(t)|^2\,dt$$

$$\therefore\ P = \sum_{n=-\infty}^{\infty} |Cn|^2$$

The average power of each phase is $P_{av} = |C_n|^2$

Example:

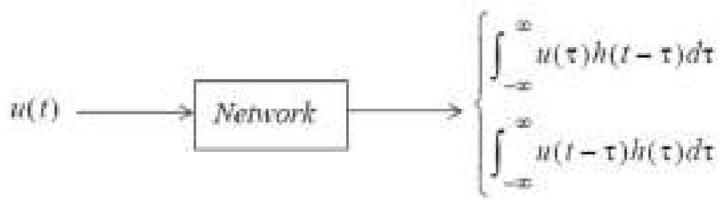
If Cn=[1 3-j4 2+j 2 2-j 3+j4 1], find P ,an energy, the instantaneous power.

Solution: $P=1^2+(3^2+4^2)+(2^2+1^2)+2^2+(2^2+1^2)+(3^2+4^2)+1^2=66wat$

Energy=P.T T: period

$$P=v^2(t)/R$$

Convolution:



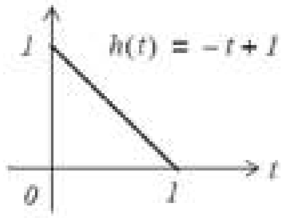
the convolution integral states that if we know the impulse response of a network, we can compute the response to any input u(t) using the integrals :

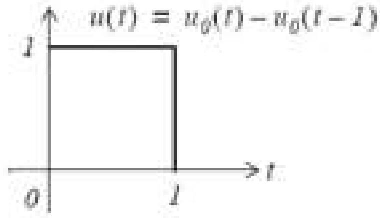
$$\boxed{\int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau} \quad \text{or} \quad \boxed{\int_{-\infty}^{\infty} u(t-\tau)h(\tau)d\tau}$$

The convolution integral is usually denoted as u(t) * h(t) or h(t) * u(t), where the asterisk (*) denotes convolution.

<u>Example:</u>

The signals u(t), and h(t) are as shown in Figure, find the convolution.





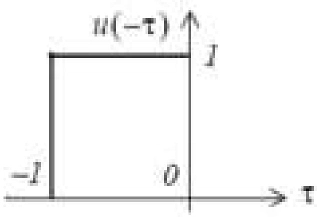
<u>Solution:</u>

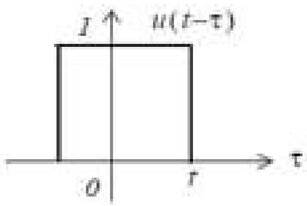
The convolution integral states that

$$h(t)*u(t) = \int_{-\infty}^{\infty} u(t-\tau)h(\tau)d\tau$$

1- we substitute u(t) and h(t) with u(τ) and h(τ).

2- We fold (form the mirror image of) u(τ) or h(τ) about the vertical axis to obtain u(-τ) or h(-τ).





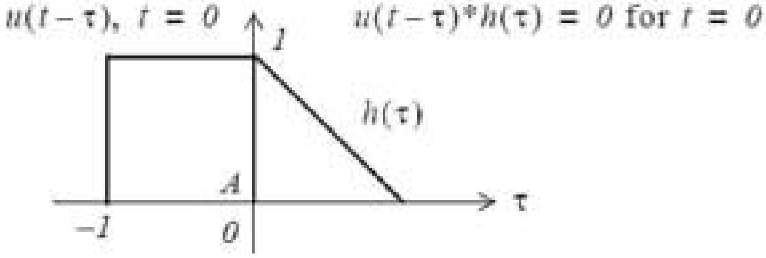
(2)Construction of u(τ)   (3) shifting u(-τ) to the right by some value t

3- we slide u(-τ) or h(-τ) to the right a distance t to obtain u(t-τ) or h(t-τ).

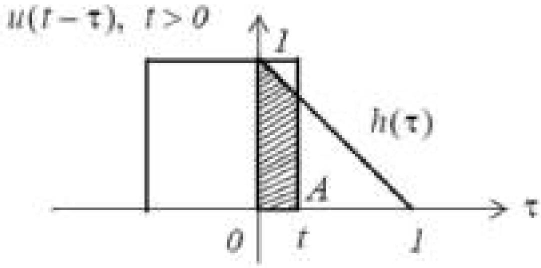
4- we multiply the two functions to obtain the product u(t-τ). h(τ), or u(τ). h(t-τ)

$$h(t)*u(t) = \int_{-\infty}^{\infty} u(t-\tau)h(\tau)d\tau$$

$= 0$ for $t=0$

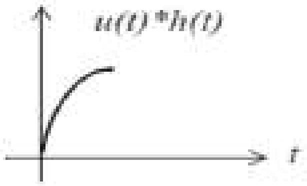


5- we integrate this product by varying t from -∞ to +∞.



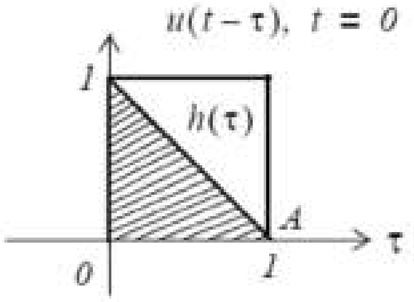
Shifting u(t- τ) to the right so that t>0

$$\int_{-\infty}^{\infty} u(t-\tau)h(\tau)d\tau = \int_{0}^{t} u(t-\tau)h(\tau)d\tau = \int_{0}^{t}(1)(-\tau+1)d\tau = \tau - \frac{\tau^2}{2}\bigg|_0^t = t - \frac{t^2}{2}$$



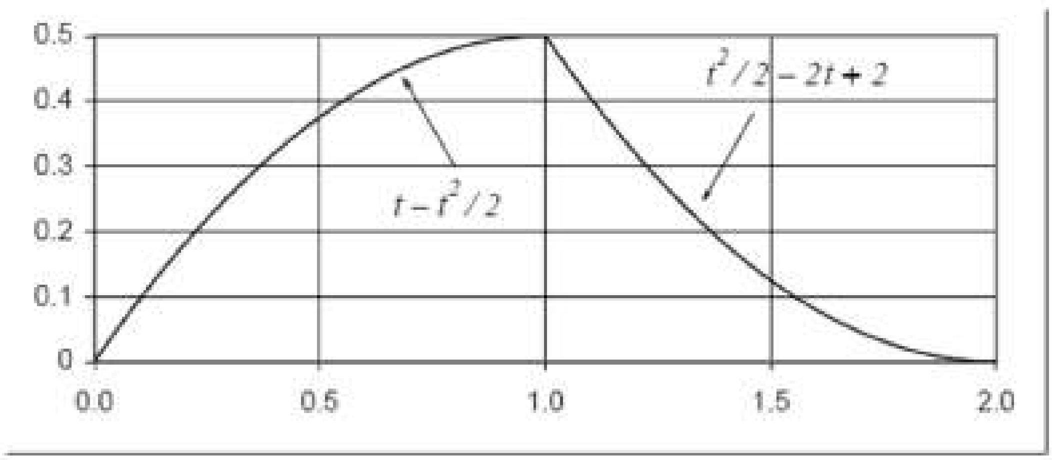
*Curve for the convolution of u(τ)*h(τ) for 0 < t < 1*

The maximum area is obtained when point A reaches t=1



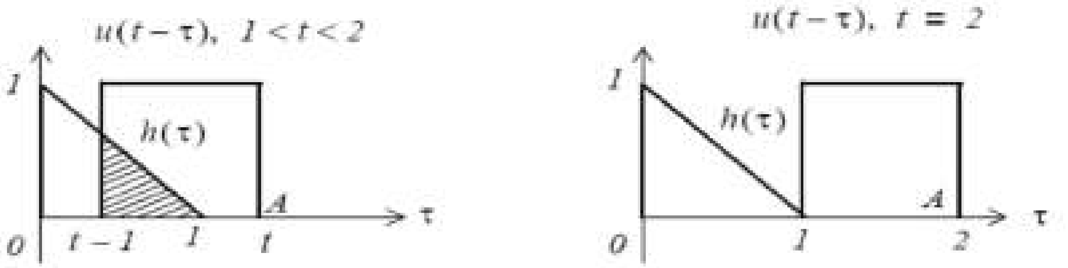
When t=1, we get

$$\left. t - \frac{t^2}{2} \right|_{t=1} = \frac{1}{2}$$



1) convolution of u(τ)* h(τ) at t=1

2) convolution of u(τ)* h(τ) at $0 \leq \tau \leq 2$



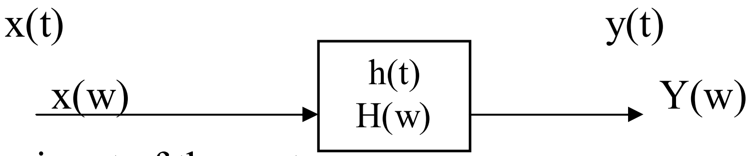
Convolution for interval 1< t < 2

$$\int_{-\infty}^{\infty} u(t-\tau)h(\tau)d\tau = \int_{t-1}^{1} u(t-\tau)h(\tau)d\tau = \int_{t-1}^{1} (1)(-\tau+1)d\tau = \tau - \frac{\tau^2}{2}\Big|_{t-1}^{1}$$

$$= 1 - \frac{1}{2} - (t-1) + \frac{t^2-2t+1}{2} = \frac{t^2}{2} - 2t + 2$$

At $t = 2$, $u(\tau)*h(\tau)=0$

Systems and transfer function



x(t) : input of the system

y(t) : output of the system (response).

h(t) : impulse response of the system.

H(w) : transfer function of the system.

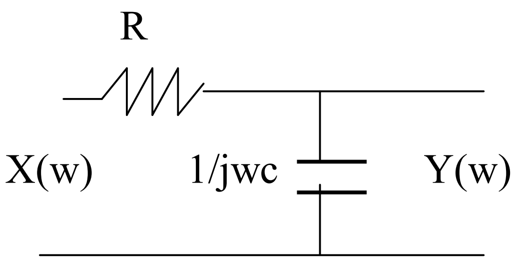
$$y(t) = x(t) \otimes \mathcal{F}^{-1} Y(w)$$

$$h(t) = y(t)\big|_{x(t)=\delta(t)} = \mathcal{F}^{-1} H(w)$$

$$H(w) = \frac{Y(w)}{x(w)}$$

<u>Example :</u>

Find the impulse response and the transfer function of the system shown below then draw |H(w)|.

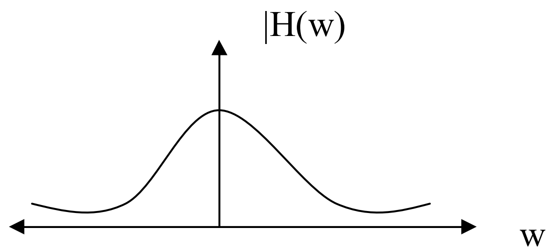


$$Y(w) = \frac{x(w)}{R + \frac{1}{jwc}} \cdot \frac{1}{jwc} = x(w)\frac{1}{1 + jwRC}$$

$$H(w) = \frac{Y(w)}{x(w)} = \frac{1}{1 + jwRC} = \left(\frac{1}{RC}\right)\frac{1}{\frac{1}{RC} + jw}$$

$$h(t) = \mathcal{F}^{-1} H(w) = \frac{1}{RC} e^{-1/RC} u(t)$$

$$|H(w)| = \frac{1}{\sqrt{1 + (wRC)^2}}$$



Filters Response :

Filters are two-port networks used to block or pass a specific rang of frequency depending on the desired characteristics, filters may be designed with RL, RC, and RLC circuits in various combinations.

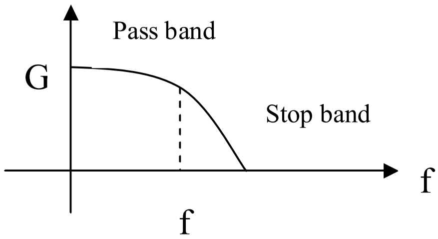
1- **low-pass filters LPF** : allows the passage of low frequencies (below fc) but block higher frequencies.

fc : the cut off frequency is the frequency which the output voltage brops below 70.7 percent of the input voltage.

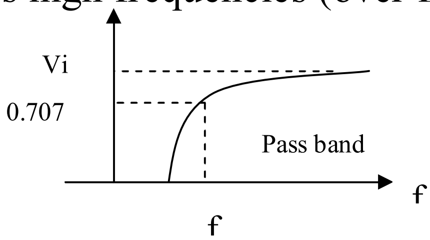
G: gain

BW=f2-f1 =fc-o=fc

∴BW=fc

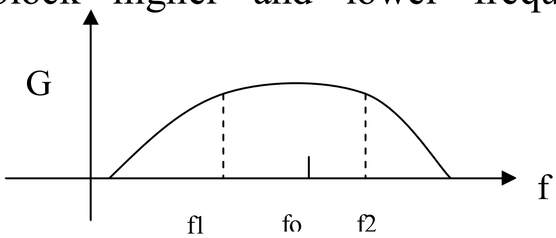


2- **high-pass filters HPF** : pass high frequencies (over fc) but block low frequencies.



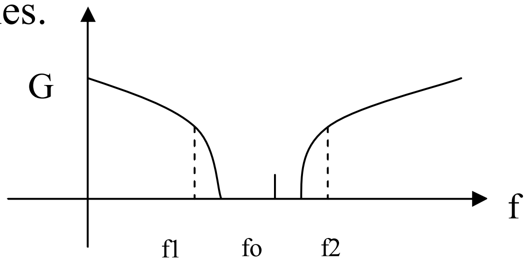
3- **band pass filters : BPF** : pass a specific rang of frequencies (within the BW) but block higher and lower frequencies. BW=f2-f1

Fo : center frequency.



4-band stop filters :

Also known as band elimination, band-rejection or band-suppression filters or wave traps, block a specific rang of frequencies (within the BW) but pass all higher and lower frequencies.



Modulation and demodulation

To transmission and reception of signal (code, voice, music, etc.) by the use of radio waves (RF) , two processes are essential

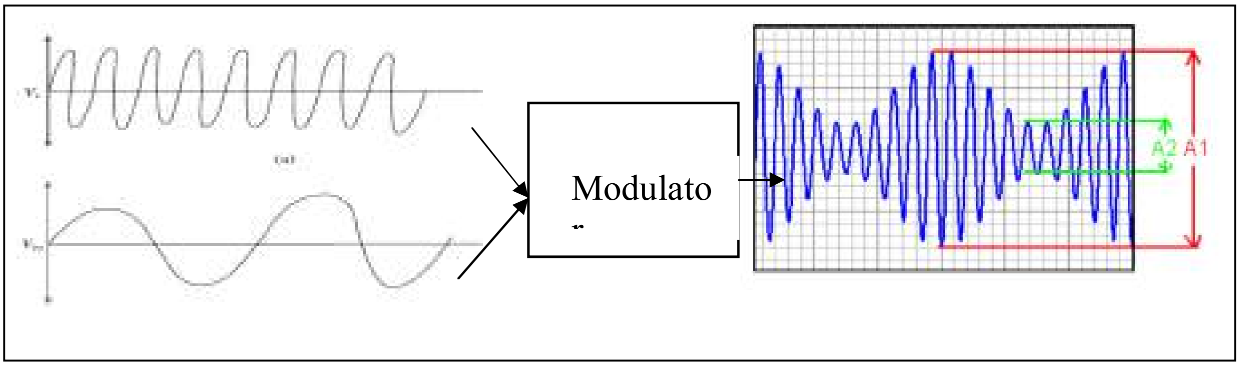
1- modulation and

2- demodulation

Modulation :

Is the processes of combining the low-frequency signal with a very high-frequency radio wave (RF) called carrier wave (CW). The resultant wave is called modulated carrier wave. (at transmitting station).

Method of modulation



A sinusoidal carrier wave is :

$$e = E_c \sin(w_c t + \phi) \qquad , w_c = 2\pi f_c$$

$$= E_c \sin(2\pi f_c t + \phi)$$

The waveform varied by three factors :

1- E_c : the amplitude

2- f_c : the frequency

3- ϕ : the phase

There are three types of modulations:

1- Amplitude modulation (AM):
Audio wave (AF) signal changes the amplitude of the carrier wave (RF) without changing its frequency or phase.

2- Frequency modulation (FM) :
Audio signal (AF) changes the frequency of the (RF) without its amplitude or phase.

3- Phase modulation (PM) :
AF changes the phase of the carrier wave without changing its other two parameters.

Anther type is **pulse modulation : -** (PAM) pulse amplitude modulation , (PPM) pulse phase modulation, (PWM) pulse width modulation.

Demodulation:

Is the process of separating or recovering the signal from the modulated carrier wave (at the receiving end).

Carrier wave : It is a high frequency undamped radio wave produced by radio- frequency (RF) oscillators (electro-magnetic waves).

Carrier waves have :

1- high- frequency waves.
2- Constant amplitude.
3- Travel with velocity of the light = $3\times10^8 m/s$
4- Their job is to carry the signal (audio- frequency AF) from transmitting to the receiving station.
5- These waves are neither seen non heard.

Notes

a- ratio frequency = 3KHz → 300GHz and none of the frequency above 300GHz is classified as radio waves.

b- Sound velocity = 345 m/s .

c- Human voice 20 → 4000 Hz

d- Audible rang 20 → 20 000 Hz

Reasons of using a carrier wave (modulation)

If we transmit the signal directly without using a carrier wave, many hurdles in the process:

1- they have relatively short- rang.

2- Noise and an interference with other transmitters operating in the same area.

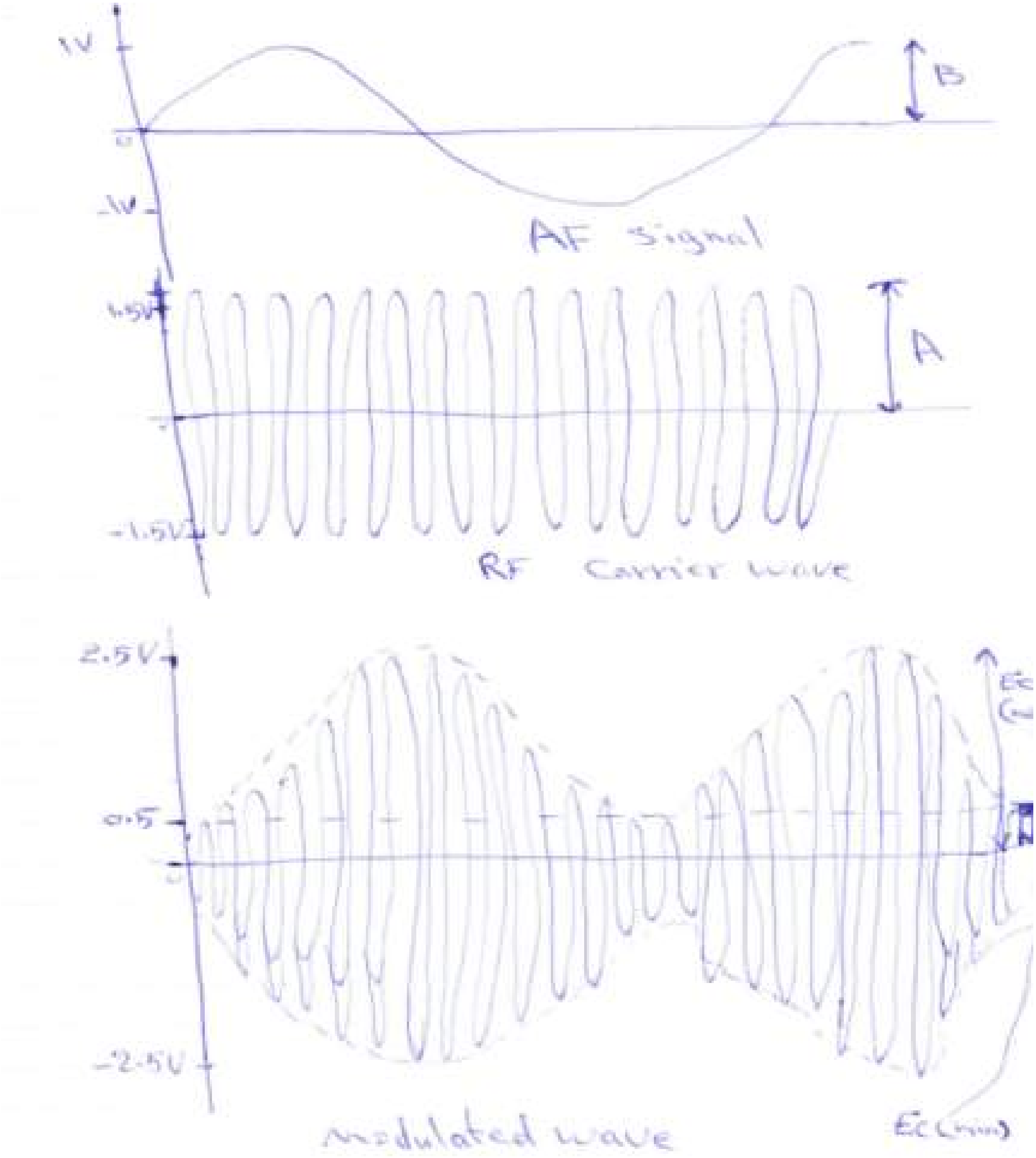
3- Efficient electromagnetic radiation requires antennas whose physical dimension are at least 1/10 λ

(λ : wave length = c / f)

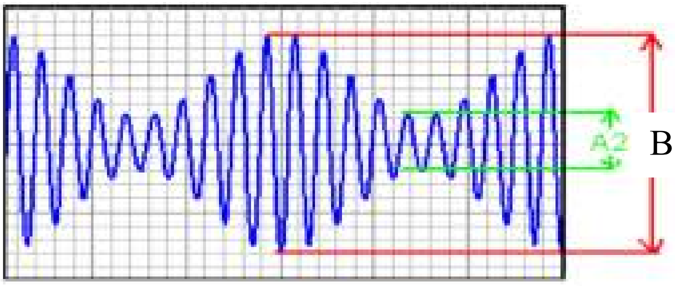
C : speed of the light = $3 \times 10^{8} m / s$

Continuous Wave Modulation

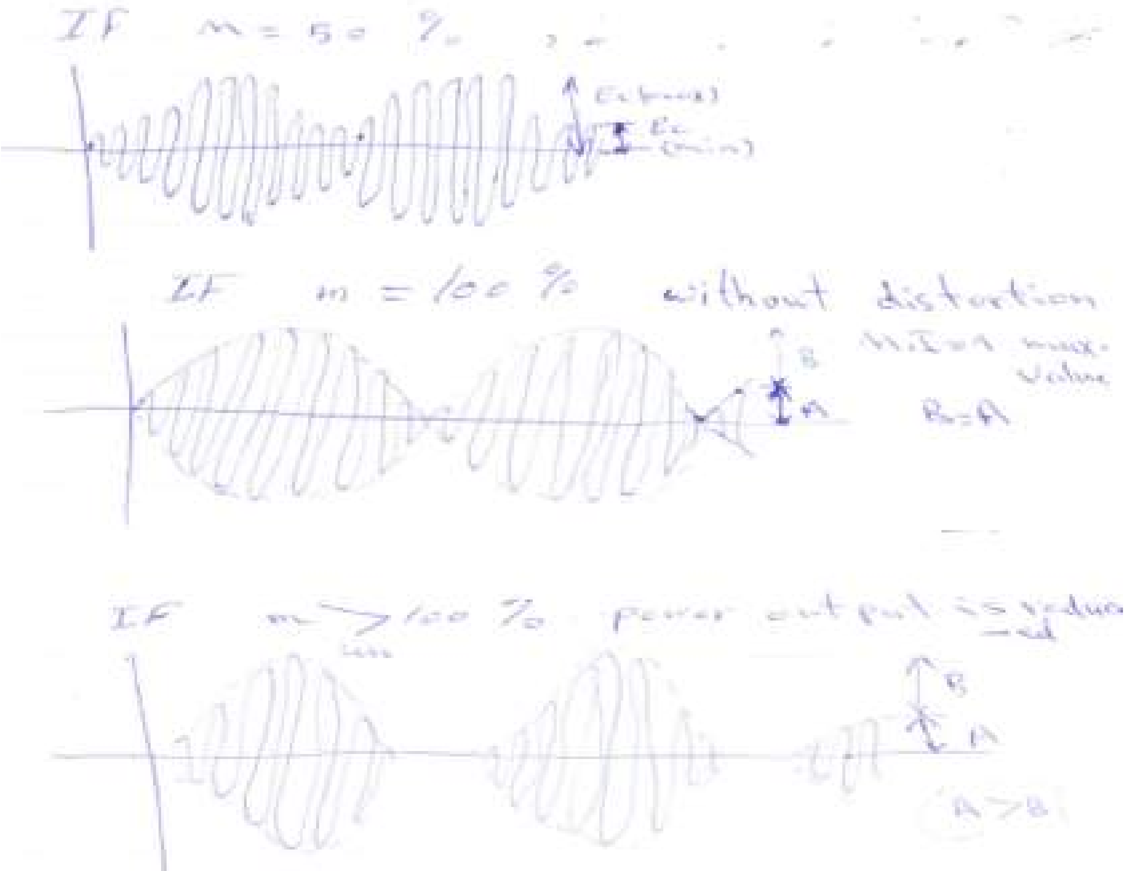
Amplitude Modulation (AM)



Percent Modulation [m]



From (0 → 100%)
m = signal amplitude / carrier amplitude
m = (B / A) × 100
Modulation index = M . I = B / A



Modulated referred to the modulated carrier wave:

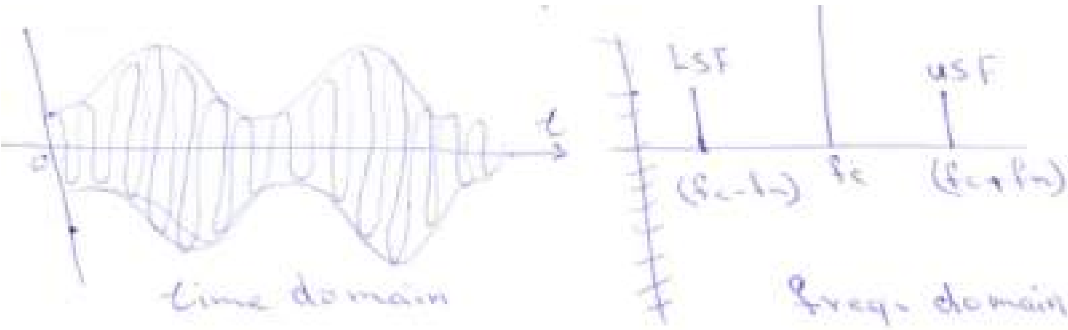
$$m = \frac{E_c(\max) - E_c(\min)}{E_c(\max) + E_c(\min)} \times 100$$

if m = 0 the amplitude of the modulating signal is zero (un modulated carrier wave). the AM wave is consist of three frequencies components:

1- f_c : the original carrier frequency component.

2- $(f_c + f_m)$: a higher frequency component (f_m a modulating Frequency signal), it called the sum component or the upper-side frequency (USF).

3- $(f_c - f_m)$ a lower frequency component, it is called the difference component or the lower side frequency (LSF).



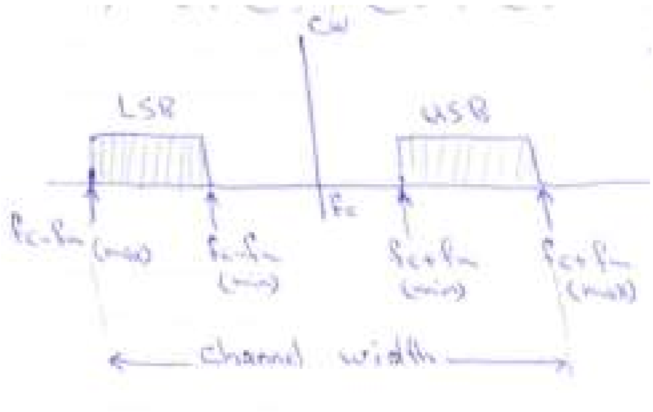
The amplitude of each side frequency = m.A / 2

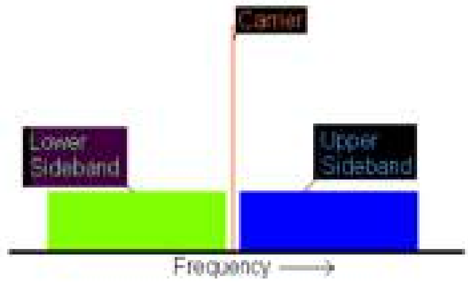
A: the amplitude of un modulated carrier wave.

LSF = USF = m.A/2

Upper and Lower Sidebands :

في محطات البث أشارة التضمين لا تضمن تردد واحد بل حزمة من الترددات.





The cannel width (or band with) is given by the difference between extreme frequencies between max. Frequency of USB and min. frequency of LSB.

Channel width = 2 × max. freq. of modulating signal

$$= 2 \times f_m \text{ (max.)}$$

Example: An audio signal given by 15sin2π (2000t) amplitude modulates a sinusoidal carrier wave 60sin2π (100000t), determine:

a) Modulation index.

b) Percent modulation.

c) Frequencies of signal and carrier.

d) Frequency spectrum of modulated wave.

Solution: B = 15 and A = 60

a) $M.I = \frac{B}{A} = \frac{15}{60} = 0.25$

b) m = M.I × 100 = 0.25 × 100 = 25%

c) f_m = 2000 Hz , f_c = 100000 Hz

d) the three frequencies present in the modulated CW are

i- 100000 Hz = 100 KHz

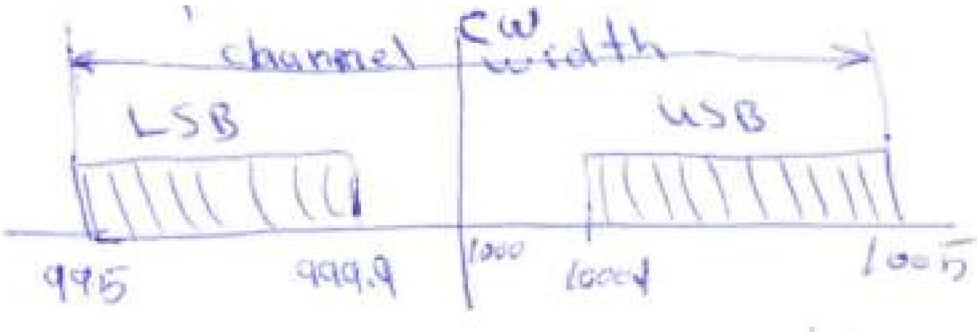
ii- 100000 + 2000 = 102000 KHz

iii- 100000 – 2000 = 98000 Hz = 98 KHz

Example:

In a broadcasting studio, a 1000 KHz carrier is modulated by an audio signal of frequency rang 100 – 5000 Hz, find (i) width or freq. rang of sidebands. (ii) maximum and minimum freq.'s of USB (iii) max. and min. freq.'s of LSB and ,(iv) width of the channel.

Solution:



(i) width of sideband = 5000 – 100 = 4900 Hz

(ii) max. freq. of USB = 1000 + 5 = 1005 KHz

min. freq. of USB = 1000 + 0.1 = 1000.1 KHz

(iii) max. freq. of LSB = 1000 – 0.1 = 999.9 KHz

min. freq. of LSB = 1000 + 5 = 995 KHz

(iv) width of channel = 1005 -995 = 10 KHz

Mathematical Analysis Of a Modulated Carrier Wave :

$$e_c = E_c \sin 2\pi f_c t = A \sin 2\pi f_c t$$
$$e_c = A \sin w_c t$$
$$e_m = E_m \sin 2\pi f_m t = B \sin 2\pi f_m t$$
$$e_m = B \sin w_m t$$

The amplitudes of the modulated carrier wave at any instant is :

$$= A + e_m$$
$$e = (A + B \sin w_m t)$$ A :constant

Instantaneous value is given by

$$e = (A + B \sin w_m t). \sin w_0 t = A \sin w_c t. + B \sin w_c t. \sin w_m t$$
$$e = A \sin w_c t + \frac{B}{2}.2 \sin w_c t. \sin w_m t$$
$$e = A \sin w_c t + \frac{B}{2}[\cos(w_c - w_m)t - \cos(w_c + w_m)t]$$
$$e = A \sin w_c t + \frac{B}{2}\cos(w_c - w_m)t - \frac{B}{2}\cos(w_c + w_m)t$$
$$e = A \sin 2\pi f_c t + \frac{B}{2}\cos 2\pi(f_c - f_m)t - \frac{B}{2}\cos 2\pi(f_c + f_m)t$$

$$2 \sin A. \sin B = \cos(A - B) - \cos(A + B)$$

where $m = B / A$, → $B=mA$

$$e = A\sin 2\pi f_c + (\frac{mA}{2})\cos 2\pi(f_c - f_m)t - (\frac{mA}{2})\cos 2\pi(f_c + f_m)t$$

it is seem that the modulated wave contains three components :

i) A sin2πfct …… the original carrier wave.

ii) mA/2 cos2π$^{(f_c + f_m)}$t …. Upper side frequency.

iii) mA/2 cos2π$^{(f_c - f_m)}$t …. Lower side frequency.

<u>Example:</u>

The tuned circuit of the oscillator in an AM transmitter uses a 40μH coil and 1nf capacitor, if the carrier wave produced by the oscillator is modulated by audio frequency up to 10KHz. Calculate the frequency band occupied by the side bands and channel width.

Solution:

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-6}\times1\times10^{-0}}} = 796KHz$$

The side band frequency 796KHz

Channel width = 2 × fm = 20 KHz

Power Relation in an AM wave

$P_c \alpha A^2 \Rightarrow P_c = KA^2$,K : constant , B=mA

USBpower, $P_{USB} \alpha (\frac{mA}{2})^2 = \frac{KB^2}{4}$

LSB power, $P_{LSB} \alpha (\frac{B}{2})^2 = \frac{KB^2}{4}$

Total sideband power $P_{sb} = 2 \times \frac{KB^2}{4} = \frac{KB^2}{2}$

Total power radiated out from the antenna is

$$P_T = P_c + P_{SB} = KA^2 + \frac{KB^2}{2}$$

$$P_T = KA^2 + \frac{K}{2}(mA)^2 = KA^2(1 + \frac{m^2}{2}),........ ..... p_c = KA^2$$

1- $P_T = P_c(1 + \frac{m^2}{2}) = P_c(\frac{2 + m^2}{2})$ If m=1 or 100% $P_T = 1.5P_c$

2- $\therefore P_c = P_T(\frac{2}{2 + m^2})$ If m=1 , $P_c = \frac{2}{3}P_T$

3- $P_{SB} = P_T - P_c = P_c(1 + \frac{m^2}{2}) - P_c$, if m=1 ,

$$P_{SB} = \frac{m^2}{2}P_c = P_T(\frac{m^2}{2 + m^2})$$

$$P_{SB} = \frac{1}{2}P_c$$

4- $P_{USB} = P_{LSB} = \frac{1}{2} P_{SB} = \frac{m^2}{4} P_c = \frac{1}{2} (\frac{m^2}{2+m^2}) P_T$ If m=1,

$$P_{USB} = P_{LSB} = \frac{1}{4} P_c$$

$$P_{USB} = \frac{1}{6} P_T$$

Example: The total power content of an AM wave is 1500w . for a 100 percent modulation determine : (1) power transmitted by carrier. (2) Power transmitted by each side-band.

Solution:

$$P_c = P_T (\frac{2}{2+m^2}) = 1500(\frac{2}{2+1}) = 1000w$$

$$P_{USB} = P_{LSB} = \frac{1}{2} P_{SB} = \frac{m^2}{4} P_c = \frac{1}{4} \times 1000 = 250w$$

Example: The total power content of an AM wave is 2.64Kw at a modulation factor of 80%, determine the power content of (i) carrier. (ii) Each side-band.

$$P_c = P_T (\frac{2}{2+m^2}) = 2.64 \times 10^3 (\frac{2}{2+0.8^2}) = 2000w$$

$$P_{USB} = P_{LSB} = \frac{1}{2} P_{SB} = \frac{m^2}{4} P_c = \frac{0.8^2}{4} \times 2000 = 320w$$

Forms Of Amplitude Modulation

في عملية التضمين AM نحصل على موجة حاملة one carrier and two side-bands , ليس من الضروري إرسال كل هذه الإشارات للحصول على الإشارة الأصلية في الاستقبال وعليه يمكن توهين او تقليل CW or side-band بدون التأثير على عملية الإرسال.

The advantages would be:

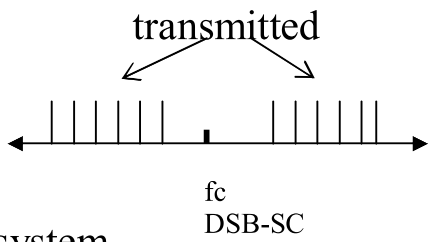
1- Less transmitted power.

2- Less band with required.

The different suppressed component system are :

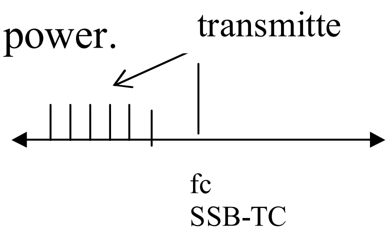
(a) DSB-SC : double side-band suppressed carrier

When m=1 carrier signal contains 66.7% for Total transmitted power.

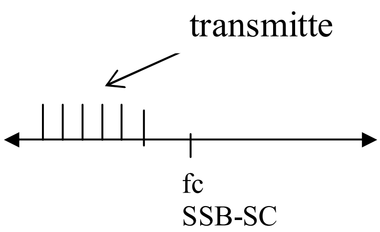


(b) SSB-TC : single sideband transmitted carrier system.

When m=1 , power saved is 1/6 of the total transmitted power.



(c) single sideband suppressed carrier system .



* in double-side band full-carrier (DSB-FC) AM, carrier conveys no information but contains maximum power.
* the two sidebands are exact images of each other they carry the same information.
* all information is available in one sideband only.
* one sideband and the carrier can be discarded with no loss of information.

Example: in an AM wave, calculate the power saving when the carrier and one sideband are suppressed corresponding to (1) m=1, (2) m=0.5.

Solution: m = 1

$$P_T = P_c(1+\frac{m^2}{2}) = 1.5P_c$$

$$P_{LSB} = P_{USB} = \frac{m^2}{4}P_c = 0.25P_c$$

$$saving = P_T - P_{SB} = 1.5P_c - 0.25P_c = 1.25P_c$$

$$\%saving = \frac{1{,}25P_c}{1.5P_c}\times 100 = 83.3\%$$

When m = 0.5

$$P_T = P_c(1+\frac{m^2}{2}) = 1.5P_c(1+\frac{0.5^2}{2}) = 1.125\ P_c$$

$$P_{LSB} = P_{USB} = \frac{m^2}{4}P_c = \frac{0.5^2}{4}P_c = 0.0625\ P_c$$

$$\therefore\ saving = \frac{1{,}125\ P_c - 0.0625\ P_c}{1.125\ P_c}\times 100 = 94.4\%$$

Amplitude modulation: suppressed carrier

The equation of a general sinusoidal signal can be written as:

$\Phi = a(t) \cos \Theta(t)$ …… modulated signal

Where:

a(t) : the time varying amplitude.

Θ : the time varying angle.

$\Theta(t) = wct + y(t)$

y(t) : the phase modulation of ϕ(t)

a(t) and y(t) are slowly varying compared to (wct), a(t) is called the envelope of the signal Φ(t) and the term wc is called the carrier frequency wc > w.

in amplitude modulation y(t)=zero (or a constant) and a(t) is made proportional to the given signal f(t) , let the constant of proportionality=1

then $\Phi(t)=f(t)\cos wct$.

coswct : the carrier signal.

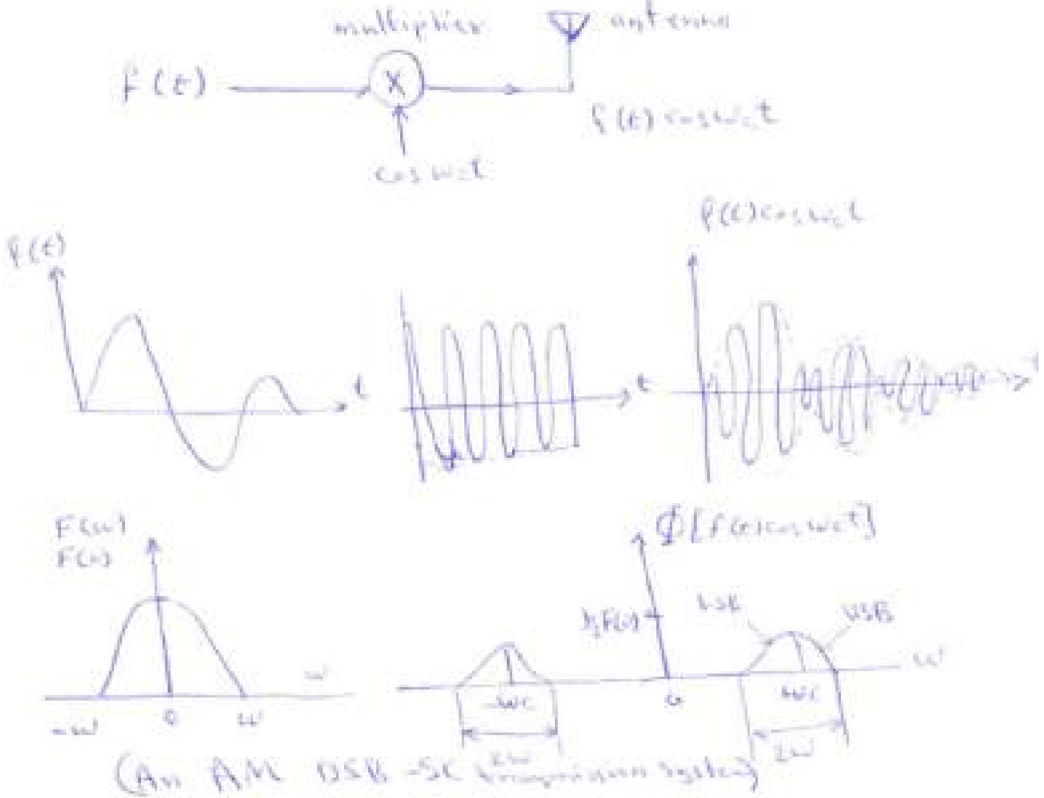
f(t) : the modulating signal.

Φ(t) : the modulated sinal.

DSB_SC

Applying the fourier transform to find the spectral density of Φ(t) is

Φ(t)=(1/2)F(w+wc)+(1/2)F(w-wc)



Amplitude modulation translates the frequency spectrum of a signal by $\pm w_c$ rad/sec but leaves the spectral shape unaltered. The spectrum is centered at the freq. wc. This type of amplitude modulation is called suppressed-carrier because the spectral density of ϕ(t) has no identifiable carrier in it.

The positive and the negative freq. content of f(t) are now displayed as positive frequencies.

Then BW=2w

For positive freq.'s the USB of ϕ(t) displays the +ve freq. component of f(t) and the LSB of ϕ(t) displays the –ve freq. components of f(t).

Demodulation or Detection

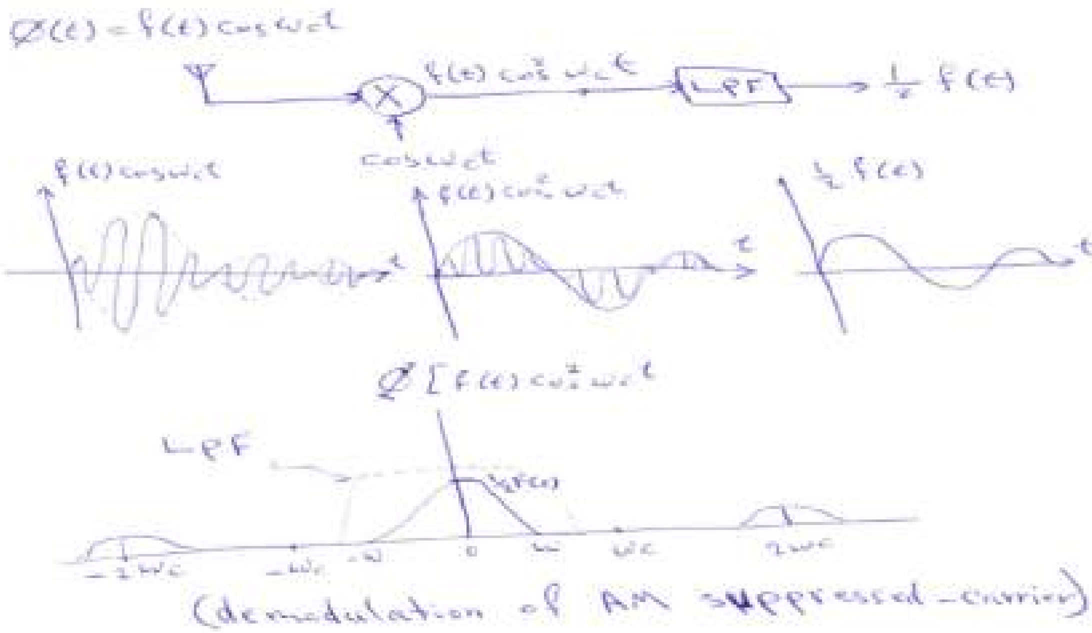
Recovery of the original signal f(t) from the DSB_SC signal ϕ(t) requires anther translation in freq. to shift the spectrum to its original position. The process of retranslation of the spectrum to its original position in freq. is called demodulation or detection.

ϕ(t)=f(t) cos wct (transmitted signal)

$$\phi(t)\cos w_c t = f(t)\cos^2 w_c t = \frac{1}{2}f(t) + \frac{1}{2}f(t)\cos 2w_c t$$

Taking the Fourier transform of both sides and using the modulating property, we get

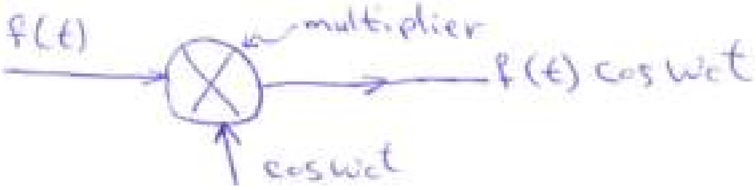
$$\phi[\varphi(t)\cos w_c t] = \frac{1}{2}F(w) + \frac{1}{4}F(w+2w_c) + \frac{1}{4}F(w-2w_c).$$



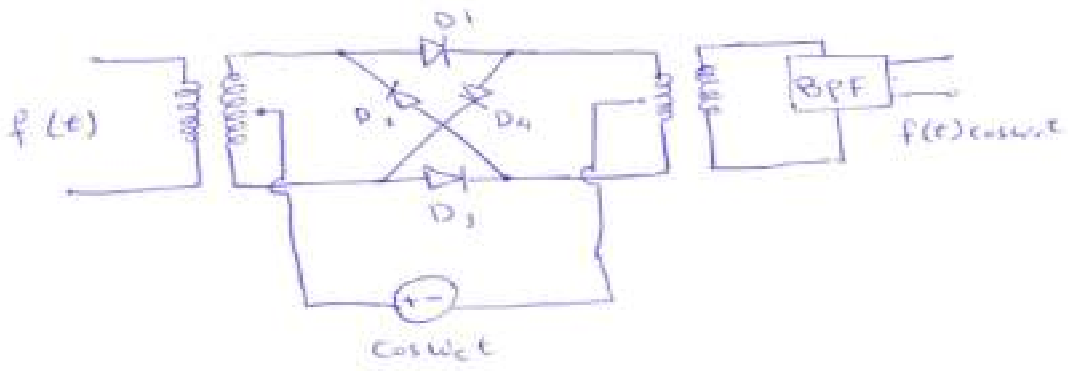
the low-pass filter is required to separate out the double freq. terms from the original spectral component. $w_c > w$

Generation of DSB-SC signals

1- **Multiplier modulators:** modulation is performed directly by multiplying f(t) by coswct using an analog multiplier.



2- the double-balanced Ring modulator



(1) in +ve half-cycles of the carrier, D1 and D3 conduct, D2 and D4 are open.

(2) In –ve halve-cycles D1 and D3 are open, and D2 and D4 are conducted.

(3) The output is proportional to f(t) during the +ve half-cycle and to –f(t) during the –ve half cycle.

Demodulation (Detection) of DSB-SC signals

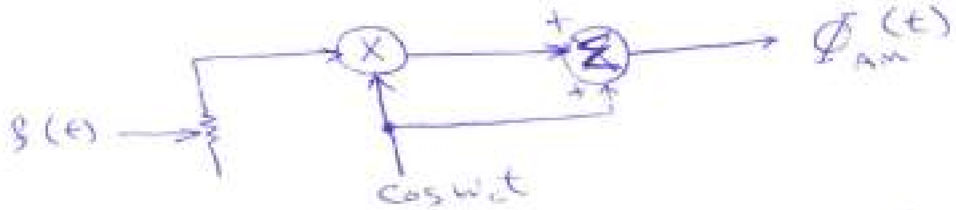
The same circuits as those used for modulation can be used for demodulation with only minor differences:-

1- in the demodulator however, the desird output spectrum is centered about w=0 and therefore a low-pass filter is needed at the output.

2- The oscillator in the demodulator must be synchronized to the oscillator in the modulator to achieve proper emodulation (synchronous detection).

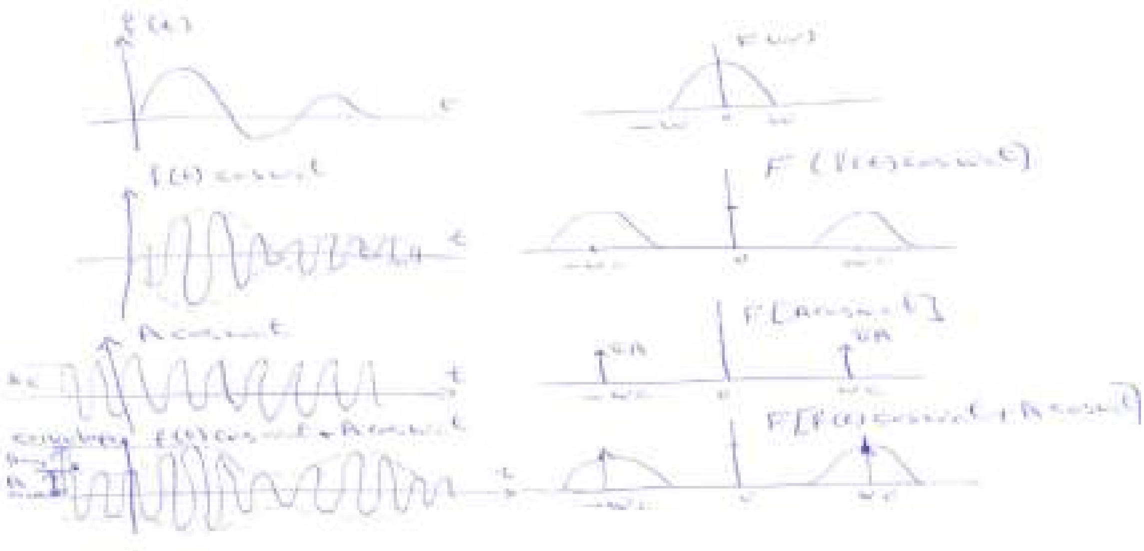
DSB-LC signals (Amplitude modulation)

The easiest way to generate a DSB-LC signal is to first generate a DSB-SC signal and then add some carrier.



$\phi(t) = f(t)\cos w_c t + A\cos w_c t$, The spectral density of $\phi(t)$ is :-

$$\phi(t) = \frac{1}{2}F(w+w_c) + \frac{1}{2}F(w-w_c) + \pi A\delta(w+w_c) + \pi A\delta(w-w_c)$$



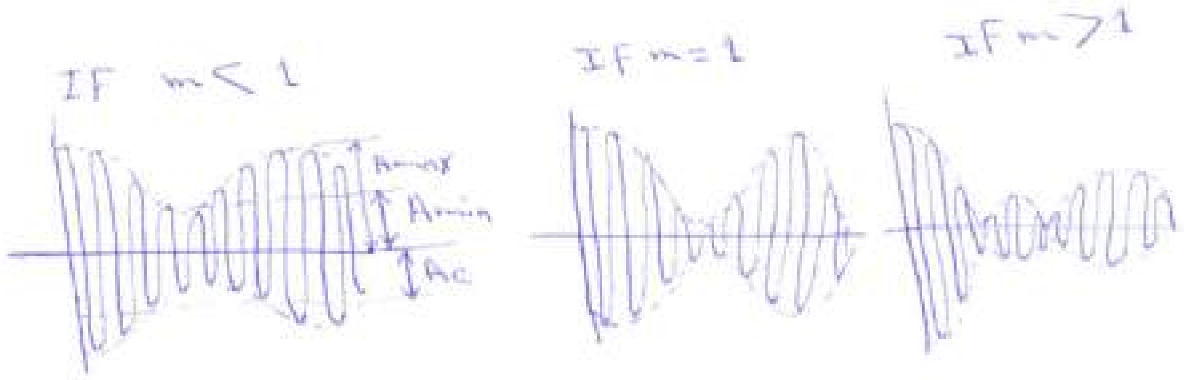
$$\phi(t) = [A + f(t)]\cos w_c t$$

[A+f(t)] : amplitude ≥ 0 at all times. This is the condition for demodulation by envelope detection.

Modulation index   m=peak DSB-SC amplitude / peak carrier amplitude

$$m = \frac{A_m}{A_c} ......or.....m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

$$A_m = \frac{A_{\max} - A_{\min}}{2},........A_c = \frac{A_{\max} + A_{\min}}{2}$$



If m > 1 , the waveform is said to be over modulated

Total power $P_t = P_c + P_s$

Transmission efficiency $\eta = \frac{m^2}{2 + m^2}$

Example: a given AM (DSB-LC) broadcast station transmits an average carrier power output of 40 kilowatts and used a modulation index of 0,707 for sine-wave modulation. Calculate (a) the total average power output. (b) The transmission efficiency.

Solution:

$$P_t = P_c(1+\frac{m^2}{2}) \quad \text{for} \quad m=0.707$$

$$P_t = (1+\frac{1}{4})P_c = \frac{5}{4}\times 40 = 50KW$$

$$\mu = \frac{m^2}{2+m^2} = \frac{(0.707)^2}{2+(0.707)^2} = 20\%$$

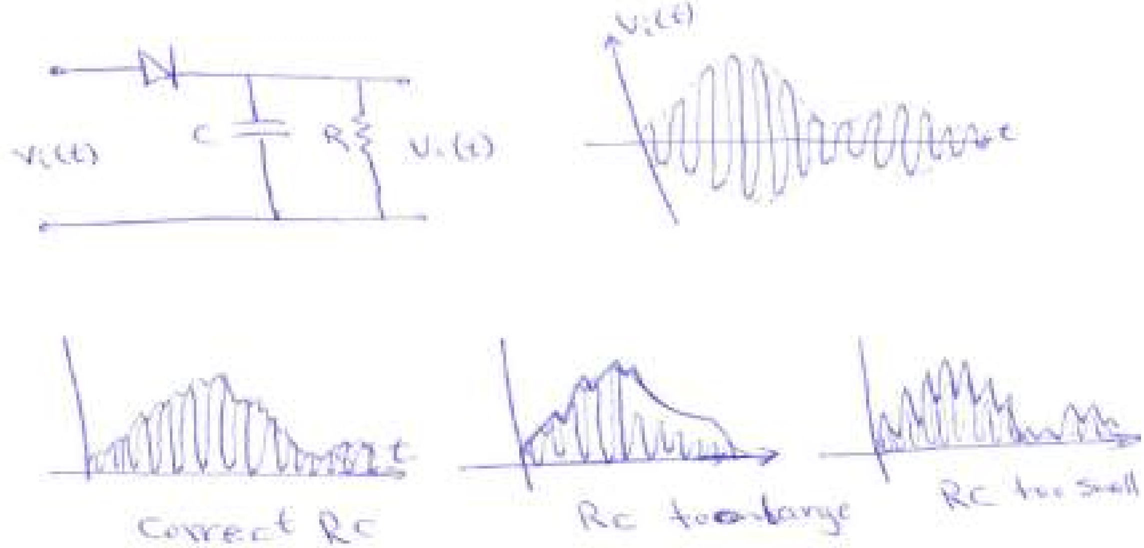
- for the (AM / DSB-SC) there is no carrier will appear,

  $P_t = P_s$ and $\eta = 100\%$. Therefore there are no losses in power.

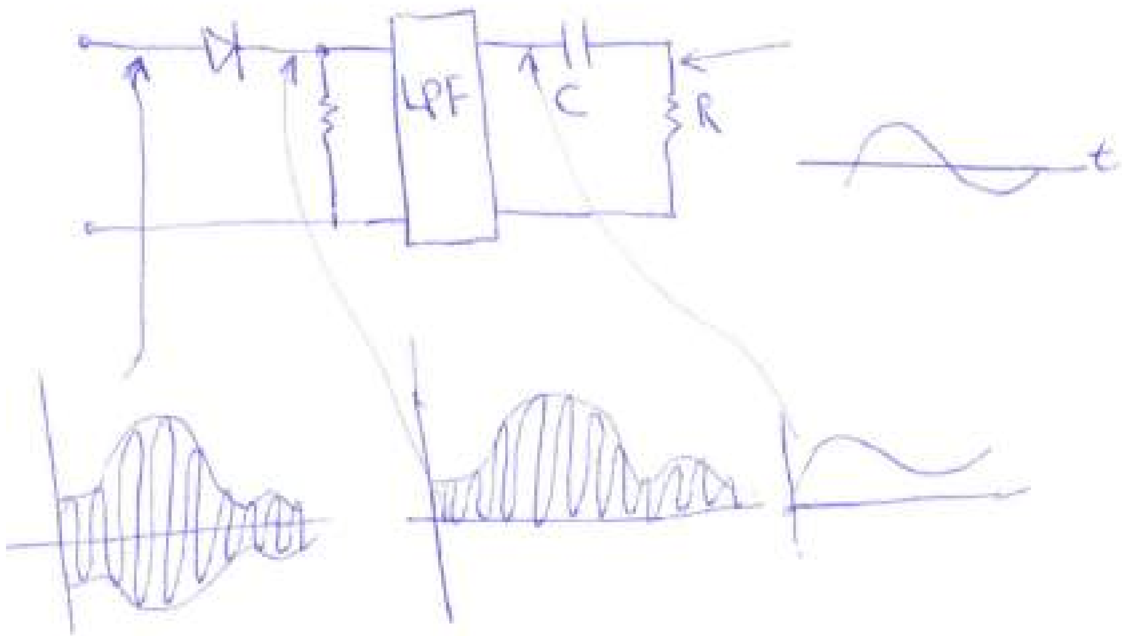
- For (AM / DSB-LC) a small portion of the total power will be in the sidebands, therefore it is necessary to increase the power of the transmission.

Demodulation (Detection) of DSB-LC (AM) signals

1) the Envelope Detector



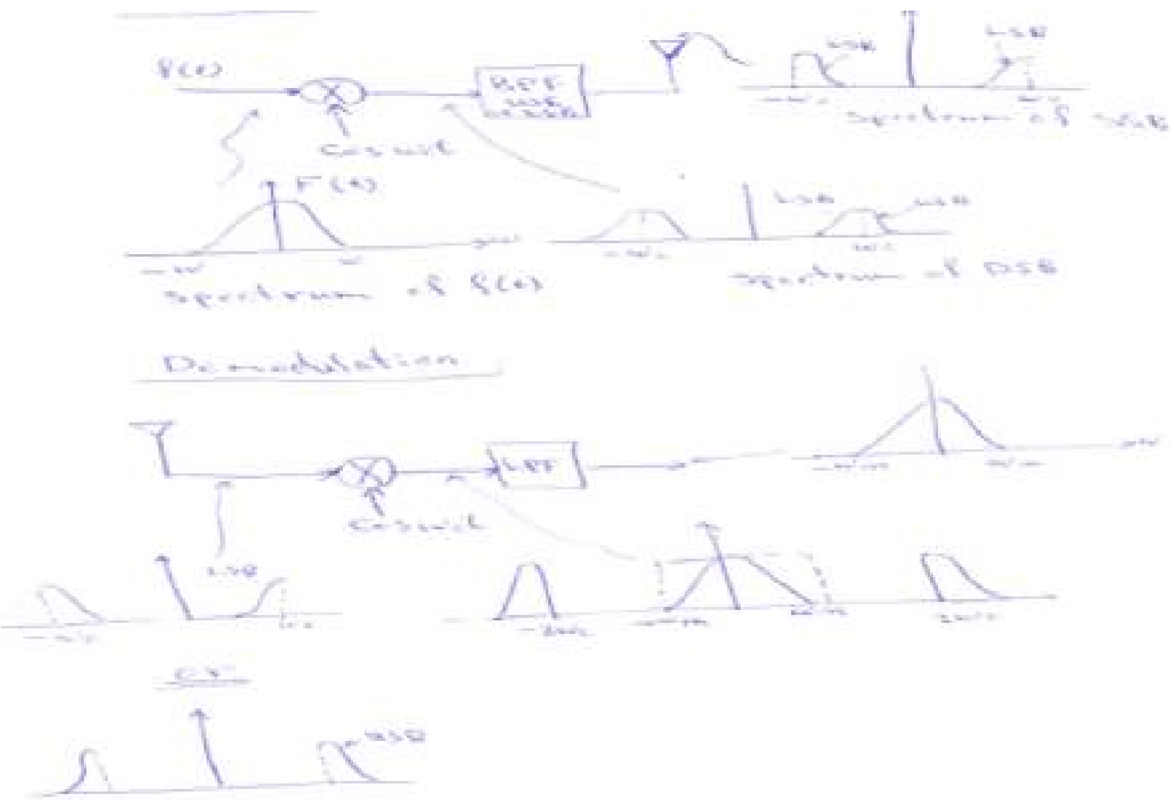
2) **The rectifier detector**



AM signal sideband suppressed carrier (SSB-SC)

The scheme in which only one sideband is transmitted is known as single sideband (SSB) transmission, and it requires only half the band with of a DSB, a SSB signal can be synchronously demodulated, multiplication of USB or (LSB) signal by [cos wct] shifts its spectrum by ±ws then the desired base band signal is obtained by using of a LPF. Hence demodulation of SSB signal is identical to that of DSB-SC signals. SSB has advantage of reducing the BW.

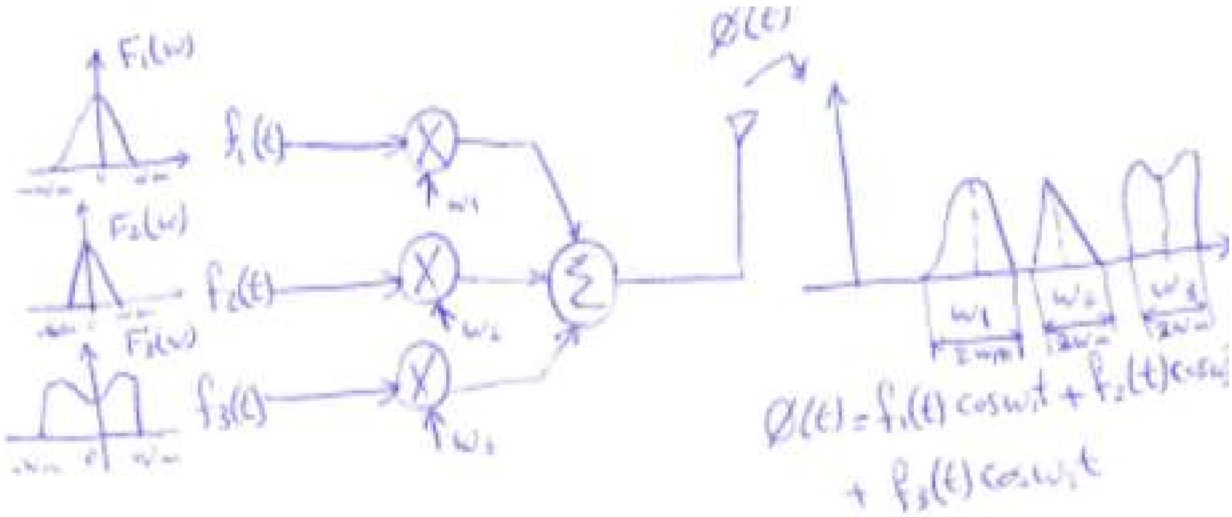
Generation of SSB signals



Frequency Division Multiplexing

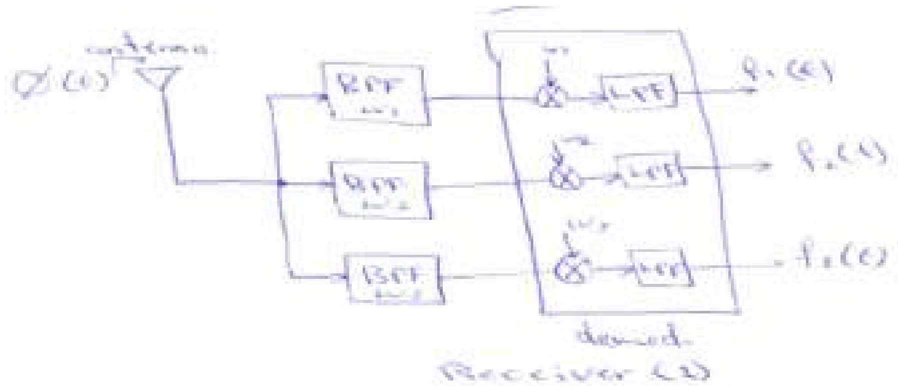
It is possible to send several signals simultaneously by choosing a different carrier frequency for each signal. These carrier frequencies are chosen so that the signal spectrum is not overlapping. This made of transmission is called frequency-division multiplexing (FDM).

We transmit several signals simultaneously using DSB-LC or DSB-SC modulation each signal is band limited to w_m rad/sec. in order to separate N signals in frequency each is modulated with a carrier frequency w1, w2, w3, ………, w_N , the spectral density of every modulation signal has a band width of 2 w_m .

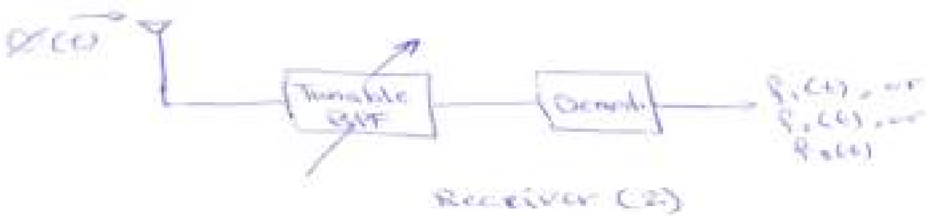


<u>At the receiver, we consider two possibilities:</u>

(1) The receiver processes the various signal spectra simultaneously, separating them in frequency with the appropriate band pass filters and then demodulating. The composite signal formed by spacing several signal in frequency may, in turn be modulated using another carrier frequency to distinguish the choice of the first carrier frequency in this case w1, w2, ….., w_N are called sub carriers.



(2) the second possibility is to have each receiver select only one of all possible signal. This is accomplished by tuning a band pass filter to the center frequency of the desired signal and then demodulating.

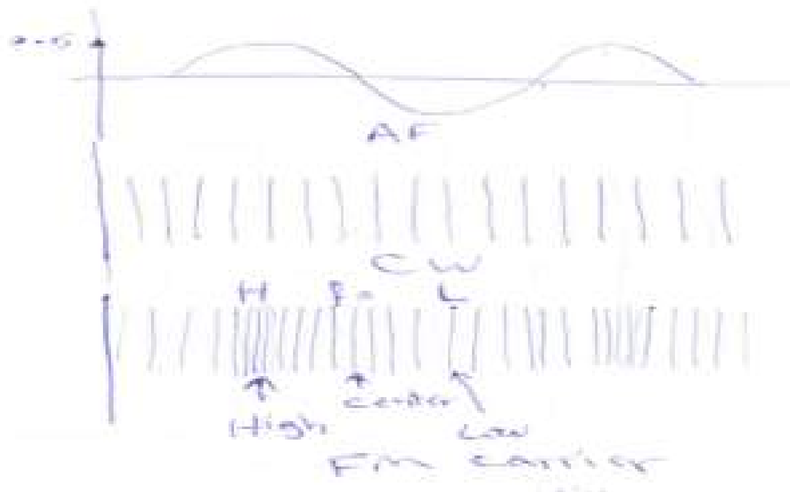


Angle Modulation

1) Frequency Modulation FM

2) Phase Modulation PM

it is modulation in which the angle of a sinusoidal (either freq. or phase) of the carrier is changed and not is amplitude. The amount of change in freq. is determined by the amplitude of modulating signal. (FM and PM) are so closely related.



H point: highest freq. when the signal amplitude is at max. Positive value.

L point: lowest freq. when AF amplitude is at max. negative value.

f_o: Normal freq. when AF amplitude is zero (resting freq. or center freq.)

Δf Freq. deviation: is the change or shift either a bove or below the resting freq..

CS carrier swing: the total variation in freq. from the lowest to the highest

CS=2×freq. deviation

CS= 2× Δf

Max. Δf = 75 KHz is allowed for FM broadcast station in the 88 to 168 MHz VHF band.

FM channel width = 2 ×75 =150 KHz allowing a 25 KHz guard band on either side:

The channel width becomes =2(75+25) =200 KHz

Modulation Index

$$m_f = \frac{freq\ .deviation}{\mathrm{mod}\ ulating\ \ .freq\ .m_f}$$

$$m_f = \frac{\Delta f}{f_m}$$

Modulation index MI can be greater than unity.

Deviation Ratio

It is the worst-case modulation index in which maximum permitted freq. deviation and max. Permitted audio freq. are used.

Then deviation ratio $= \dfrac{(\Delta f)(\max.)}{f_m(\max.)}$

For FM broadcast station (Δf)max. = 75 KHz , and max. permitted freq. of modulating audio signal is 15 KHz.

Then: deviation ratio $= \dfrac{75KHz}{15KHZ} = 5$

For sound portion of commercial TV , deviation ratio $= \dfrac{25KHz}{15KHZ} = 1.67$

Percent modulation

It is given by the ratio of actual freq. deviation to the max. allowed deviation.

$$m = \frac{(\Delta f)_{actual}}{(\Delta f)_{\max.}}$$

If m= 100% then (Δf)actual = (Δf)max.

If (Δf)actual = 50 KHz, m=50/75=2/3 = 0.667 = 66.7%

If m= 0 then un modulated carrier wave

Example: what is the modulation index of an FM carrier having a carrier swing of 100 KHz and a modulating signal of 5 KHz?

Solution:

CS = 2 × Δf

Δf = CS / 2 = 100 / 2 =50KHz

$$m_f = \frac{\Delta f}{f_m} = \frac{50}{5} = 10$$

Example: an FM transmission has a freq. deviation of 18.75 KHz. Calculate percent modulation if it is broadcast (i) in the 88→108 MHz band. (ii) as a portion of TV broadcast.

Solution:

(i) for this transmission band (Δf)max.= 75 KHz

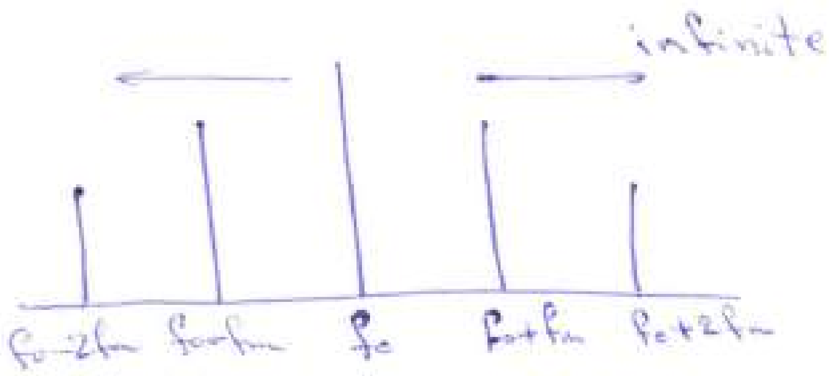
m = 18.75 / 75 × 100 = 25%

(ii) in the case (Δf)max = 25 KHz

M = 18.75 / 25 = 75 %

FM sidebands

In FM a number of sidebands are formed (infinite), (in AM two side freq.'s), they lie on both sides of f_o at equal distances have equal amplitudes.



FM carrier contains the following freq.'s:

i) f_o ii) $f_o \pm f_m$ iii) $f_o \pm 2f_m$ iv) $f_o \pm 3f_m$ and so on

(their strength becomes negligible after a few sidebands)

The bandwidth BW = 2n f_m ,n: is the highest order of significant sideband.

BW $= 2(1+m_f)f_m$

When the sideband having amplitudes less than 5 % or when m_f is least

6: BW $= 2(1+m_f)f_m$, $m_f = \frac{\Delta f}{f_m}$ $\rightarrow BW = 2(\Delta f + f_m)$

The number of sideband is depending:

1- Directly on the amplitude of the modulating signal.

2- Inversely on the freq. of the modulating signal.

Example: in an FM circuit, the modulation index is 10 and the highest modulation freq. is 20 KHz. What is the approximate bandwidth of the resultant FM signal?

Solution: since the value of m_f is more than 6

Then $BW = 2(\Delta f + f_m)$, $m_f = \frac{\Delta f}{f_m}$ $\rightarrow$ 10=Δf / 20

Δf = 200 KHz

BW = 2(200+20)=440KHZ

Mathematical Expression

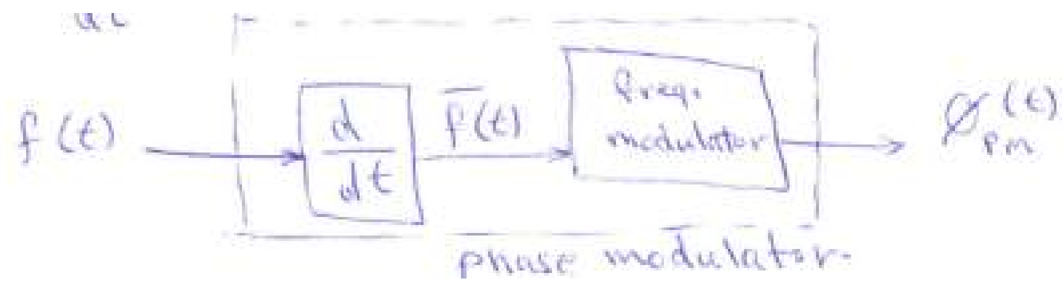
For PM $\phi_{PM}(t) = A_c \cos(w_c t + K_P f(t))$

$K_P f(t) = \phi_c$, K_P : Constant

$\therefore \theta(t) = w_c t + K_P f(t)$

The instantaneous freq. $w_i(t) = \frac{d\theta}{dt}$

$$\frac{d\theta}{dt} = w_c + K_P \overline{f(t)}$$

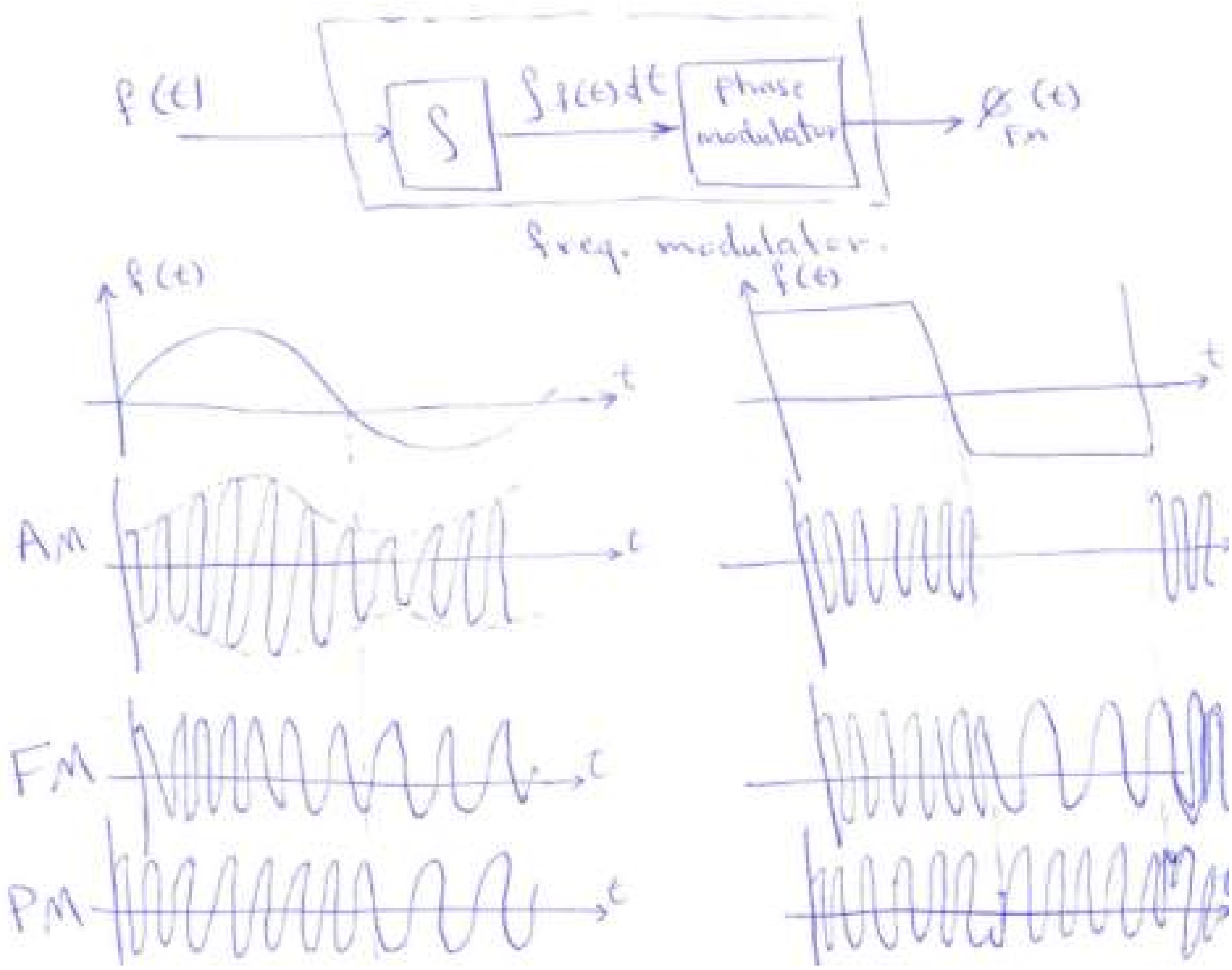


For FM $\phi_{FM}(t) = A_c \cos(w_c t + K_f f(t))t$, K_f: Constant

$$\theta(t) = [w_c + K_f f(t)]t$$

$$\theta(t) = \int w_i(t)dt = \int_{-\infty}^{t} (w_c + K_f f(t))dt$$

The angle $\theta(t) = w_c t + K_f \int_{-\infty}^{t} f(t)dt$



Example: Determine the instantaneous freq. of the signal $\phi(t) = A\cos(10\pi t + \pi t^2)$.

Solution:

$$\theta(t) = 10\pi t + \pi t^2$$

$$w_i(t) = \frac{d\theta}{dt} = 10\pi + 2\pi t = 2\pi(5+t)$$

$$P_{angle-\text{mod.}} = \frac{A_c^{\,2}}{2}$$

Ac: the amplitude is always constant, hence the power is constant.

Generation of Angle-Modulated Wave

The generation of FM:

$$\phi_{FM}(t) = A_c \cos(w_c t + K_f f(t)t)$$

$$\phi_{FM}(t) = A_c \cos(w_c + K_f f(t))t$$

$$\theta_i = w_c + K_f f(t)$$

$$f(t) = A_m \cos w_m t$$

$$\theta_i = \int_0^t w_i dt = \int_0^t (w_c + K_f A_m \cos w_m t)dt \text{ ,......... .} w_i = w_c + K_f f(t) = w_c + K_f A_m \cos w_m t$$

$$\theta_i = w_c t + \frac{K_f A_m}{w_m} \sin w_m t$$

$$\frac{K_f A_m}{w_m} = \frac{\Delta f}{f_m} = m_f \text{ : modulation} - index \text{ ,......... ...} K_f A_m = \Delta f$$

$$\therefore \theta_i = w_c t + m_f \sin w_m t$$

$$\therefore \phi_{FM}(t) = A_c \cos(w_c t + m_f \sin w_m t)$$

If $m_f << 1$ is called narrow band FM (NBFM).

If $m_f >> 1$ is called wide band FM (WBFM).

Generation of (NBFM) and (NBPM)

$$\phi_{FM}(t) = A_c \cos(w_c t + m_f \sin w_m t)$$

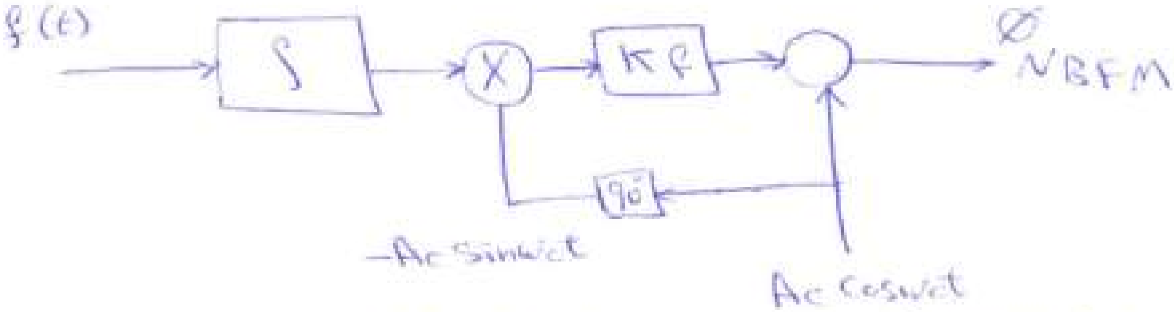
$$\phi_{FM}(t) = A_c \cos w_c t \cos(m_f \sin w_m t) - A_c \sin w_c t \sin w_c t \sin(m_f \sin w_m t)$$

$$IF.....m_f << 1 \Rightarrow \cos(m_f \sin w_m t) \cong 1$$

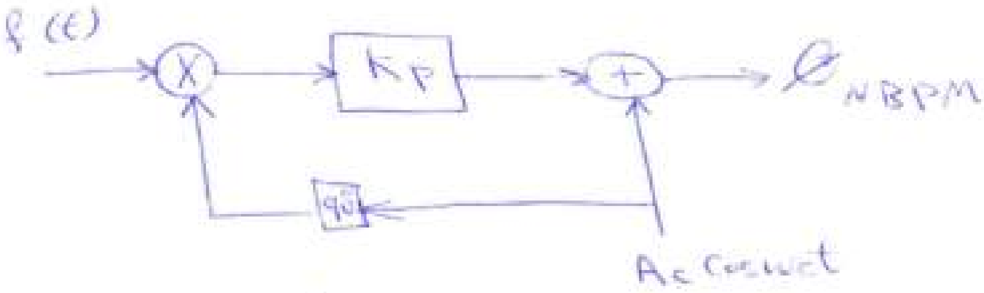
$$..........................\sin(m_f \sin w_m t) \cong m_f \sin w_m t$$

$$\phi_{NBFM}(t) = A_c \cos w_c t - A_c m_f \sin w_c t \sin w_m t$$

$$\phi_{NBFM}(t) = A_c \cos w_c t - \frac{A_c m_f}{2}(\cos(w_c - w_m)t - \cos(w_c + w_m)t).$$



block diagram of (NBFM)



(NBPM)

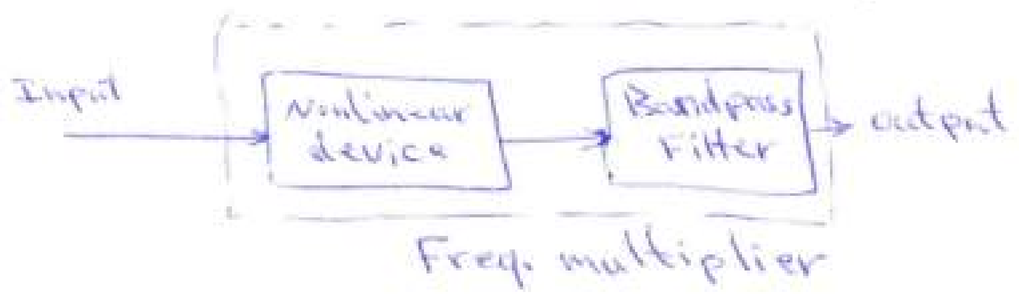
Generation of WBFM

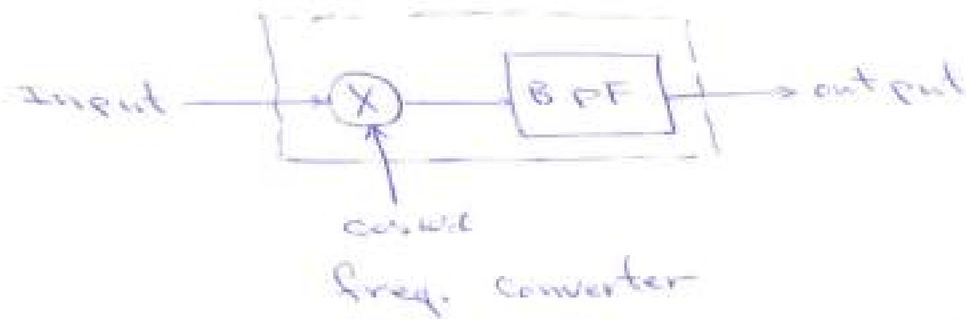
m_f is large $\gg 1$

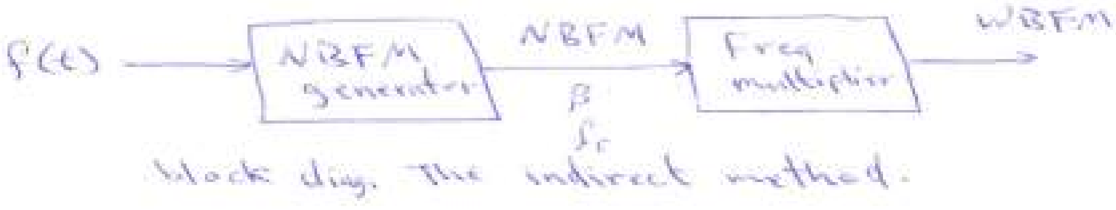
There are two ways of generating WBFM:

(1) The indirect method

We convert NBFM to WBFM by using frequency multiplier which increases the carrier freq. of FM waveform and the modulation index. This is a disadvantage can be solved by using a frequency converter.

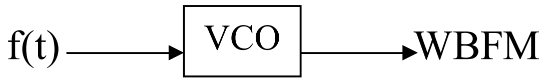






(2) The direct method

A voltage-controlled oscillator (VCO) is used to generate WBFM signal. The VCO is an oscillator whose the oscillation freq. can be controlled by an input control voltage.



The BW of FM and PM

$BW = 2f_m(m_f + 1)$ Carson's rule

In NBFM $m_f << 1$

BW = 2 f_m Δw (rad/s), Δf (Hz)

$$m_f = \frac{\Delta w}{w_m} = \frac{\Delta f}{f_m}$$

$$m_{f_{PM}} = K_P A_m \Rightarrow (\Delta w)_{PM} = K_P A_m w_m$$

$$m_f = \frac{K_f A_m}{w_m} \Rightarrow \Delta w_{(FM)} = K_f A_m$$

FM equation: $\phi(t) = A_c \cos(w_c t + A_m K_f \sin w_m t)t$

PM equation: $\phi(t) = A_c \cos(w_c t + A_m K_P \sin w_m t)$

Example: a given FM signal is $\phi_{FM} = 10\cos(10^6 \pi t + 8\sin 10^3 \pi t)$, determine (a) the carrier freq. (b) the modulating index. (c) the peak freq. deviation. (d) the BW. (e) is the signal NBFM or WBFM. (f) the average power.

Solution:

$$(a) w_c = 2\pi f_c \Rightarrow f_c = \frac{10^6 \pi}{2\pi} = 500 \ KHz$$

$$(b) m_f = K_f A_m = 8$$

$$(c) m_f = \frac{\Delta f}{f_m} \Rightarrow \Delta f = m_f . f_m = 8 \times 500 = 4 KHz$$

$$(d) BW = 2 f_m (m_f + 1) = 2 \times 500 \ (8 + 1) = 9 KHz$$

$$(e) m_f >> 1 .... \therefore WBFM$$

$$(f) \frac{A_c^2}{2} = \frac{100}{2} = 50 \ watt$$

Demodulation of FM

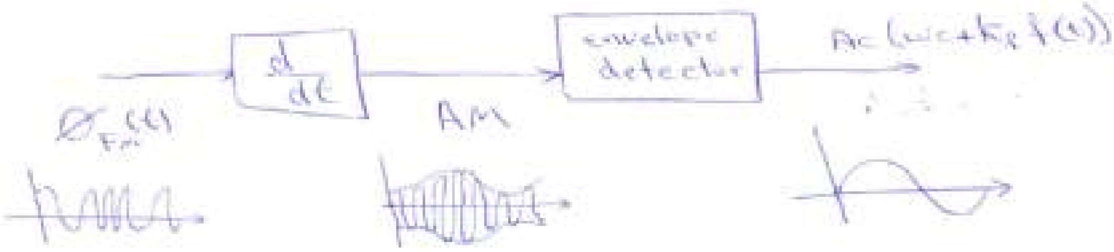
(1) A system which is used to detect FM signal is called (frequency discriminator).

$$\phi_{FM} = A_c \cos(w_c t + K_f \int_0^t f(t) dt)$$

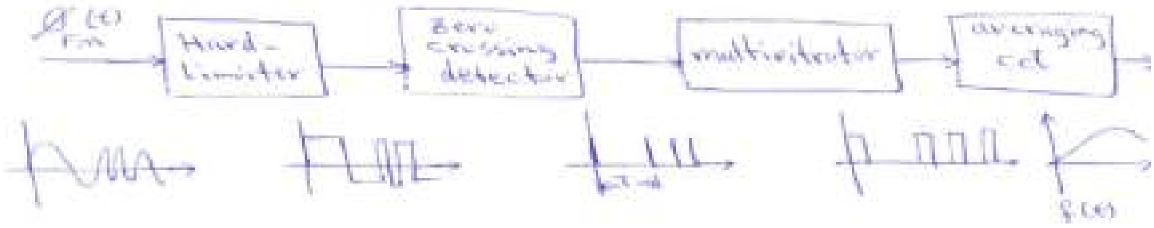
$$\int_0^t f(t)dt \quad \frac{d\phi_{FM}}{dt} = \underbrace{-A_c(w_c t + K_f f(t)}_{A_m} \underbrace{sin(w_c t + K_f \int_0^t f(t)dt)}_{F_m}$$

* $\frac{d}{dt}$ (differentiator used to changed FM into AM signal.

*AM signal detected by an envelope detector



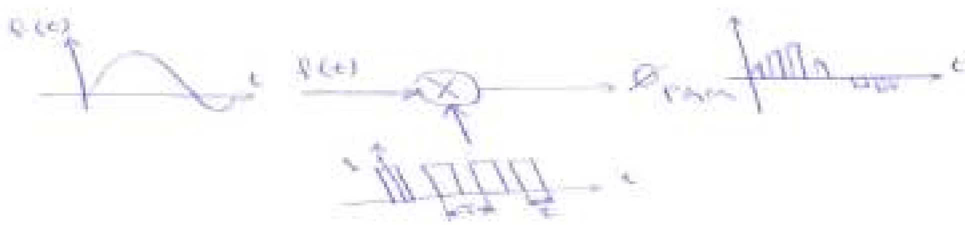
(2) Zero Crossing detector:-



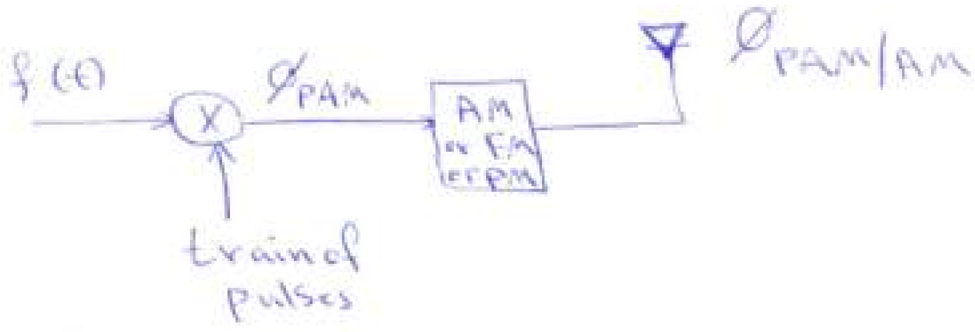
Pulse Modulation

(1) pulse Amplitude Modulation (PAM):

In pulse-amplitude modulation (PAM) the amplitude of a train of constant-width pulses is varied in proportion to the sample values of the modulating signal. The pulses are usually taken at equally spaced intervals of time.



The pulse-modulation waveform may be transmitted directly on a pair of wire lines. But for long distance transmission, it is more suitable to use higher freq.'s carriers with CW modulation. We can used DSB or FM, and PM in (PAM):

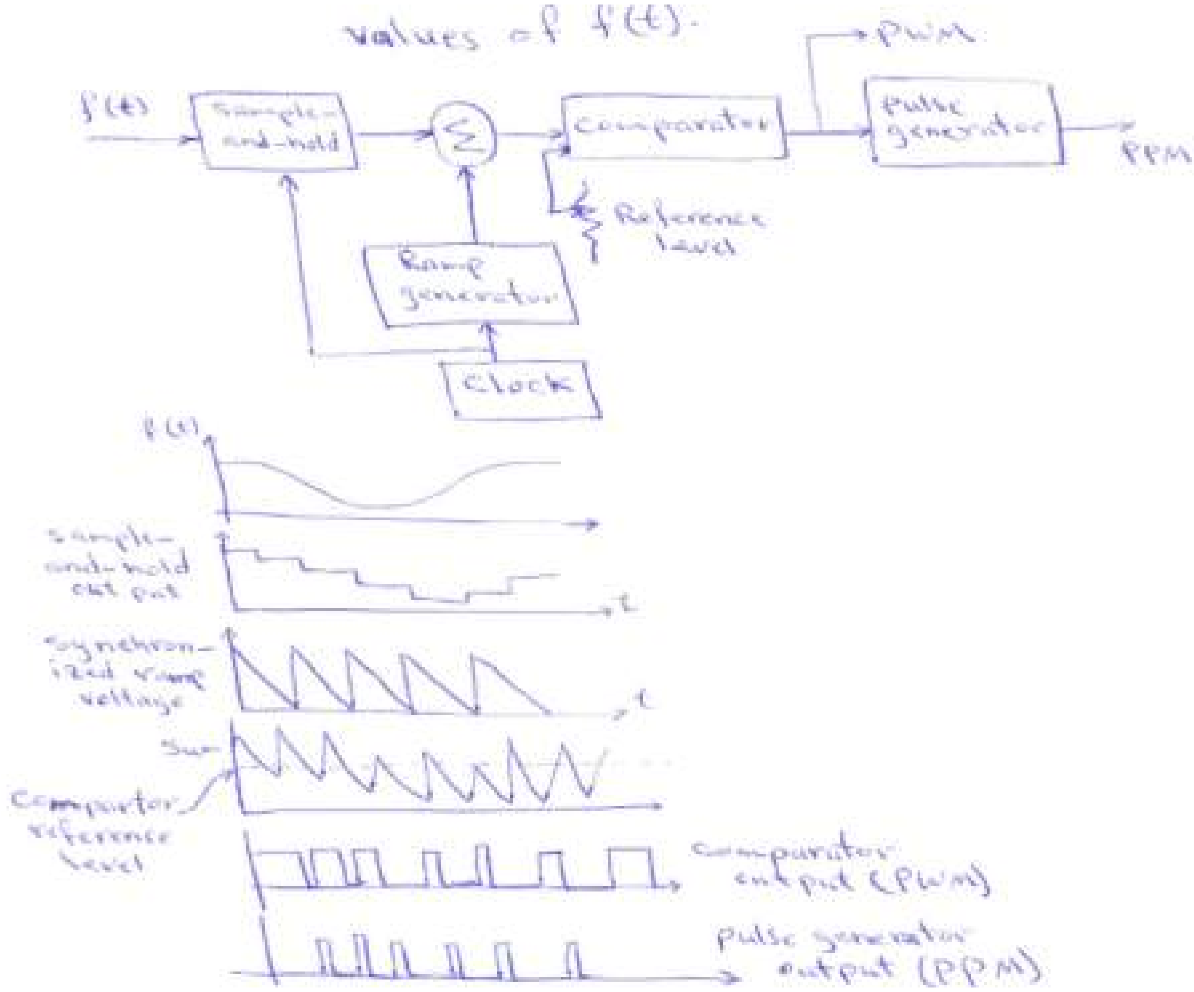


(2) <u>Pulse-width (PWM) and pulse-position (PPM) modulation:</u>

a- PWM: the width of a train is constant amplitude pulses vary in proportion to f(t).

b- PPM: keep both the amplitude and width of the pulses constant but vary the pulses positions in proportion to the values of f(t).

values of f(t).


PWM
f(t)
sample-and-hold
Σ
Comparator
pulse generator
PPM
Reference level
Ramp generator
Clock
f(t)
sample-and-hold out put
synchronized ramp voltage
Sum
Comparator reference level
comparator output (PWM)
pulse generator output (PPM)

Comparison between AM and FM

1- All transmitted power in FM is useful where as in AM most of it is in carrier which serves on useful purpose.
2- FM has high signal-to-noise (S/N) ratio. It is due to two reasons: (1) there happens to be less noise at VHF band. (2) FM receivers are fitted with amplitude limiters which remove amplitude variations caused by noise.
3- In FM due to guard-band there is hardly any adjacent channel interference.
4- Since only transmitter freq. is modulated in FM only fraction of a watt of audio power is required to produce 100 % modulation as compared to high power required in AM.

FM has the following disadvantages:

1- It requires much wider- channel almost 7 to 15 times as large as needed by AM.
2- It requires complex and expensive transmitting and receiving equipment.
3- Since FM reception is limited to only line of sight, area of reception for FM is much smaller than for AM.

Application for FM transmission:

1- use it in FM broadcast 88-108 MHz with 200 KHz channel in which commercial FM stations broadcast programmers to their listeners.
2- Use it in TV through video signal is amplitude modulated, sound is transmitter which is freq. modulated.
3- Use is in the mobile or emergency services which transmit voice freq.'s (20-4000) Hz only.
4- Use it in the amateur bands where a gain only voice freq.'s are transmitted.