

Figure (5.10). Typical stepper performance curve

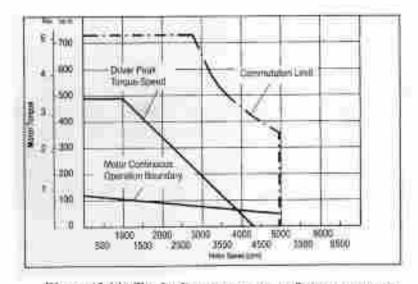


Figure (5.11) Typical servemeter performance curve

Some of the important questions a designer may need to consider when choosing a motor for an application include the following:

# Motor starting torque and motor acceleration

The torque at zero speed, called the starting torque, is the torque the motor can deliver when rotation begins. For the system to be self-starting, the motor must generate torque sufficient to overcome friction and any load torques. The acceleration of the motor and load at any instant is given by:

$$\alpha = \frac{(T_{motor} - T_{tond})}{1}.$$

Where  $\alpha$  is the angular acceleration in rad/sec<sup>2</sup>;  $T_{motor}$  is the torque produced by the motor;  $T_{loud}$  is the torque dissipated by the load; Jis the total polar moment of inertia of the motor rotor and the load. The difference between motor and load torques determines the acceleration of the system. When the motor torque is equal to the load torque, the system is at a steady state operating speed.

## Maximum speed

The zero torque point on the torque-speed curve determines the maximum speed a motor can reach. Note that the motor cannot deliver any torque to the load at this speed. When the motor is loaded, the maximum no-load speed cannot be achieved.

## Operating duty cycle

When a motor is not operated continuously, one must consider the operating cycle of the system. The duty cycle is defined as the ratio of the time the motor is on with respect to the total elapsed time. If a load requires a low duty cycle, a lower-power motor may be selected that can operate above rated levels but still perform adequately without overheating during repeated on-off cycles.

# Load power requirement

The power rating is a very important specification for a motor. Knowing the power requirements of the load, a designer should choose a motor with adequate power based on the duty cycle.

### Power source

Whether the motor is AC or DC might be a critical decision. Also, if battery power is to be used, the battery characteristics must match the load requirements.

### Load inertia

As the Equation of acceleration  $\alpha$  implies, for fast dynamic response, it is desirable to have low motor rotor and load inertia J.When the load inertia is large, the only way to achieve high acceleration is to size the motor so it can produce much larger torques than the load requires.

## Constant speed to drive load

The simplest method to achieve constant speed is to select an AC synchronous motor or a DC shunt motor which runs at a relatively constant speed over a significant range of load torques.

## Accurate position or speed control required

In the cases of angular positioning at discrete locations and incremental motion, a stepper motor is a good choice. A stepper motor is easily rotated to and held at discrete positions. It also can rotate at a wide range of speeds by controlling the step rate. The stepper motor can be operated with open-loop control, where no sensor feedback is required. However, if you attempt to drive a stepper motor at too fast a step rate or if the load torque is too large, the stepper motor may slip and not execute the number of steps expected. Therefore, a feedback sensor such as an encoder might be included with a stepper motor to check if the motor has achieved the desired motion.

For some complex motion requirements, where precise position or speed profiles are required (for example in automation applications where machines need to perform prescribed programmed motion), a servomotor may be the best choice. A servomotor is a DC, AC, or brushless DC motor combined with a position sensing device (digital encoder). The servomotor is driven by a programmable controller that processes the sensor input and generates amplified voltages and currents to the motor to achieve specified motion profiles. This is called closed-loop control, since it includes sensor feedback. A servomotor is typically more expensive than a stepper motor, but it can have a much faster response.

# Transmission and gearbox

Often loads require low speeds and large torques. Since motors usually have better performance at high speed and low torque, a speed-reducing transmission (gear box or belt drive) is often needed to match the motor output to the load requirements.

The term gear motor is used to refer to a motor-gearbox assembly sold as a single package. When a transmission is used, the effective inertia of the load is:

$$J_{eff} = J_{load} \left( \frac{\omega_{load}}{\omega_{motor}} \right)^2$$
.

where  $I_{eff}$  is the effective polar moment of inertia of the load as seen by the motor. The sum of this inertia and the motor rotor inertia can be used in the Equation tocalculate acceleration. The speed ratio  $(\frac{\omega_{load}}{\omega_{motor}})$  is called the gear ratio of the transmission. It is often specified as a ratio of two numbers, where one or both numbers are integers (which is always the case when using meshing gears, which have integer numbers of teeth). So a gear ratio is sometimes written (N:M), which can be read as: N to M gear reduction. This means N turns of the motor are required to create M turns of the load, so for an N:M gear ratio, the speeds are related by:

$$\omega_{load} = (M/N)\omega_{motor}$$

# Motor torque-speed curve and the load

If the load has a well-defined torque-speed relation, called a load line, it is wise to select a motor with a similar torque-speed characteristic. If this is the case, the motor torque can match the load torque over a large range of speeds, and the speed can be controlled easily by making small changes in voltage to the motor.

# Operating speed for a known motor torque-speed curve and load line

As shown in Figure (5.12), a given motor torque-speed curve and a well-defined load line, the system settles at a fixed speed operating point. Furthermore, the operating point is self-regulating. At lower speeds, the motor torque exceeds the load torque and the system accelerates toward the operating point, but at higher speeds, the load torque exceeds the motor torque, reducing the speed toward the operating point. The operating speed can be actively changed by adjusting the voltage supplied to the motor, which in turn changes the torque-speed characteristic of the motor.

### Motor reverse

Some motors are not reversible due to their construction and control electronics, and care must be exercised when selecting a motor for an application that requires rotation in two directions.

## Size and weight restrictions

Motors can be large and heavy, and designers need to be aware of this early in the design phase.

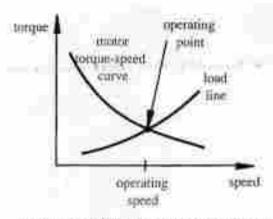


Figure (5.12). Motor operating speed

## 4.3. GENEVA DRIVE

The Geneva drive is a gear mechanism that translates a continuous rotation into an intermittent rotary motion. The rotating drive wheel has a pin that reaches into a slot of the driven wheel advancing it by one step. The drive wheel also has a raised circular blocking disc that locks the driven wheel in position between steps.

A geneva mechanism is used to drive an indexing table intermittently. The drive mechanism is shown in Figures (5.13a, 5.13b and 5.14) of an indexing table. Note that there are two wheels: a driver and the driven indexing table. The actual driving force is delivered by a pin on the driver which slides in an arc-shaped slot in the driven indexing table. The driver rotates continuously, imparting motion to the table while passing through angle B of its motion (Figure (5.15)), During the remainder of the revolution of the driver, the driven indexing table is at rest. This at-rest (or dwell) period is the time during which work is accomplished at each work station of the indexing table.

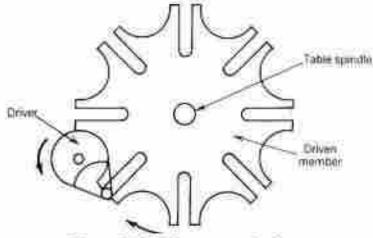


Figure (5.13a). Geneva mechanism

In Figure (5.4a), note that the path of the driver pin is precisely radial (orthogonal to tangential) to the driven wheel both at the times it enters and exits the slot. This is necessary to impart a smooth start and complete stop to the indexing table. In fact, any coasting of the table during the station execution (wheel-at-rest) phase will cause the driver pin to miss its next entry slot. This is a serious problem that will jam the drive for the machine and produce other mishaps that may occur on top of the table. Note also that the index and dwell time are dependent upon the driver speed and are constant from cycle to cycle.



Figure (5.13b), Geneva mechanism

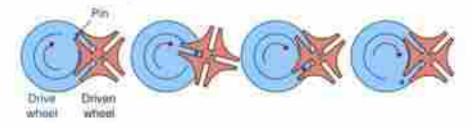


Figure (5.14) Geneva mechanism rotation

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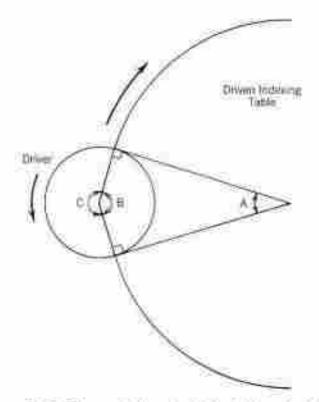


Figure (5.15). Geneva driven indexing table calculations

It is possible to have various numbers of stations in a geneva indexing table setup, although the number cannot be less than three. More than eight stations should be considered rare. Figure 5.15 describes the relationships between number of stations, dwell time, index time, and driver speed. Note that at four stations angle A – angle B, and driver and driven wheel diameters are the same. An example will now illustrate calculations for a typical geneva.

Angle A + Angle B = 
$$180^{\circ}$$

Angle B + Angle C =  $360^{\circ}$ 

Index time =  $\frac{\text{Angle B}}{360^{\circ}} \times \frac{1}{\text{driver rpm}}$  (minutes)

Dwell time =  $\frac{\text{Angle C}}{360^{\circ}} \times \frac{1}{\text{driver rpm}}$  (minutes)

Cycle time = Index time + Dwell time =  $\frac{1}{\text{driver rpm}}$ .

### MECHANIZATION OF PARTS HANDLING

## PARTS FEEDING

Fabricated piece parts, whether they are forgings, castings, machined parts, plastic injection moldings, wood milled products, electrical components, or rubber moldings, must be:

- Transported.
- 2. Selected.
- Oriented properly.
- Positioned for assembly or subsequent operations.

# 1.1. Parts Source Compatibility

When automating the assembly station, the first place the automation engineer should look is where the components were before they came to the assembly station. The engineer should ask, "Was it really necessary for the components to arrive for assembly at this station all jumbled and piled into a hopper?" Even if the components were purchased outside the plant, it is sometimes permissible to specify that they be shipped in magazines or continuous feed strips, or at least compartmentalized containers.

# 1.2. Motion and Transfer

If the worst case is indeed the actual case and the parts are received in a jumbled mass in a hopper, the first requisite for automatic feeding of these parts is to get them to start moving.

Motion can be imparted to parts by gravity, centrifugal force, tumbling, air pressure, or vibration. The power to drive these mechanisms can be either electrical or off-machine, the off-machine devices having synchronization advantages.

Off-machine drive: Automation engineers employ separate electric motors to feed a machine automatically in situations in which an off-machine linkage would be cheaper and more practical. Furthermore, there is an advantage of off-machine linkages that is synchronization. When a machine speed is increased, the kinematic linkages attached to it speed up right along with the machine.

The most versatile small parts feeding machine is the electrically driven vibratory bowl (Figure (6.1a and 6.1b). The most amazing feature of a vibratory bowl is that it literally causes piece parts to travel uphill as they vibrate up inclined ledges, or tracks, spiraling around the inside of the bowl. The magic creating this phenomenon is the variation in the acceleration of motion during the vibration cycle causing the parts to be tossed upward farther than they slip backward before the next vibration cycle tosses them farther upward. The direction of the vibration motion is canted to provide two components, one parallel to travel in the track and one perpendicular to the track. The frequency of vibration can be varied, with increasing frequency resulting in faster feed rates up to a limit beyond which increasing frequency results in slower feed rates.



Figure (6.1a). Vibratory bowl feeder



Figure (6.1b). Vibratory bowl feeder

Another type of vibratories the vibratory spiral elevator, shown in Figure (6.2). The spiral elevator has some advantages when the objective is strictly feeding, not orienting or rejecting misoriented parts. This type usually used for special applications like in a heat chamber to bring electronic parts to the required temperature during their fifty-six-foot spiral travel up the track before being discharged into a test operation.

Three ways to feed cylindrical parts are shown in Figures (6.3a, 6.3b and 6.3c). The centerboard hopper uses a reciprocating motion. The other two use rotation, but for different purposes. The centrifugal hopper uses the rotation to provide centrifugal force, while the rotary disk feeder merely uses it to tumble the parts around until they fall by gravity into one of the orientation slots.

For more gentle feeding of delicate cylindrical parts, a rotating base hopper can be used as shown in Figure (6.4).

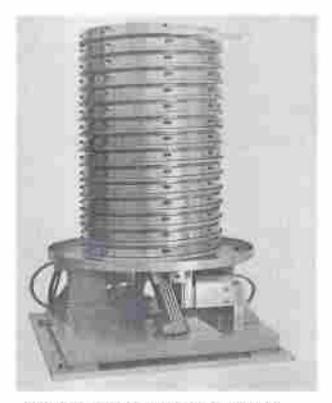


Figure (6.2). Vibratory spiral elevator

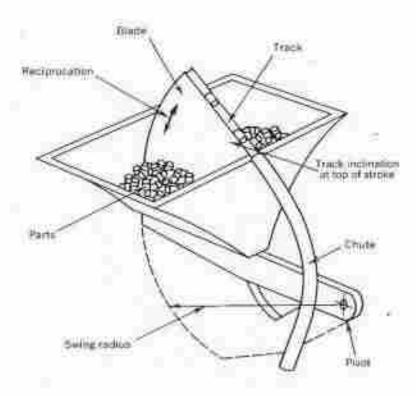


Figure (6.3a). Three ways to feed cylindrical parts, (a) Centerboard hopper

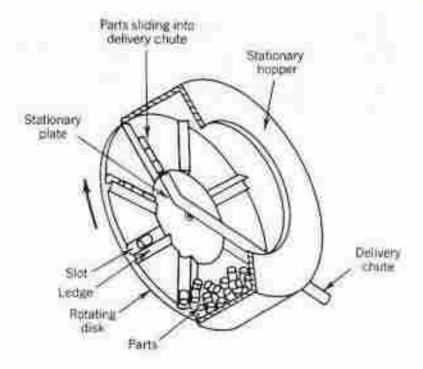


Figure (6.3b). Three ways to feed cylindrical parts, (b) Rotary disk feeder

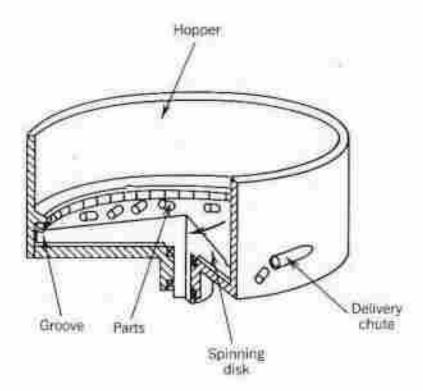


Figure (6.3c). Three ways to feed cylindrical parts, (c) Centrifugal hopper

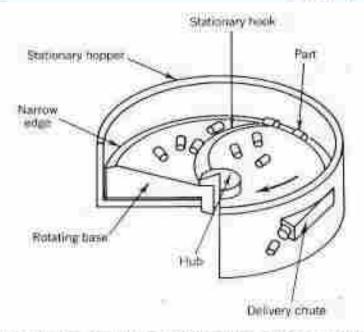


Figure (6.4) Rotating base hopper for gentle feeding of delicate cylindrical parts

Two more feeders for cylindrical parts, the bladed wheel hopper and the tumbling barrel hopper, are shown in Figures (6.5a and 6.5b). The bladed wheel bears a resemblance to the centerboard hopper, but both feeders are really quite different. The bladed wheel does not capture parts; it merely stirs up the mass and kicks misaligned parts back up out of the way for unimpeded passage of those parts that have fallen into the slot. The radius of the bladed wheel is such that it does not touch parts that have already correctly fallen into the slot. The bladed wheel hopper is essentially a gravity feed device. The tumbling barrel, like a clothes dryer, has vanes around the drum to pick up parts as the drum rotates so that the parts will fall upon the track in the middle. Too much rotation speed will obviously defeat the purpose of this device.

The rotary centerboard hopper (Figure (6.6)) appears similar to the bladed wheel hopper, but note in the figure that the blade is actually picking up the parts, not kicking them back. The part is U-shaped and travels down and around by gravity on the slowly turning blade to the point at which it is dropped upon the stationary delivery track.

For disk-shaped parts, a revolving hook hopper can be used (Figure (6.7)). The appearance is similar to the rotating base hopper of Figure (6.4), but note that the base in Figure (6.7) is stationary. The purposes are quite different, however, as the revolving hook, which is of less thickness than the disks, slides along the floor of the hopper.

gathering the disks into a channel that drops them into the center of the hopper. If the disks are ferrous, they may be fed using a magnetic feeder. Figures (6.8a and 6.8b) shows two types of magnetic feeders.

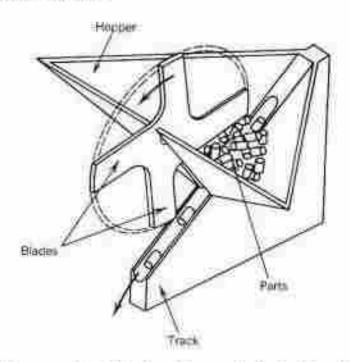


Figure (6.5a). Bladed wheel hopper and tumbling barrel hopper for the feeding of durable cylindrical parts, (a) Bladed wheel hopper

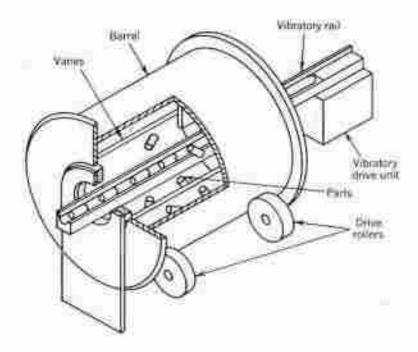


Figure (6.5b). Bladed wheel hopper and tumbling barrel hopper for the feeding of durable cylindrical parts, (b) Tumbling barrel hopper

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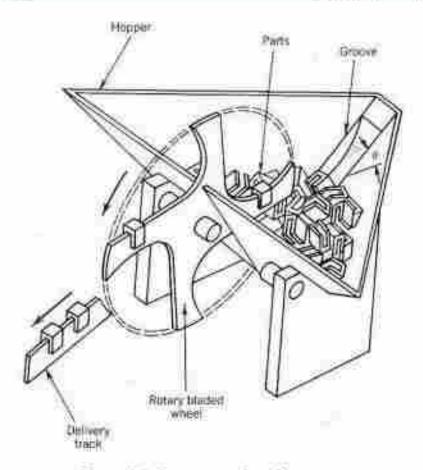


Figure (6.6). Rotary centerboard hopper

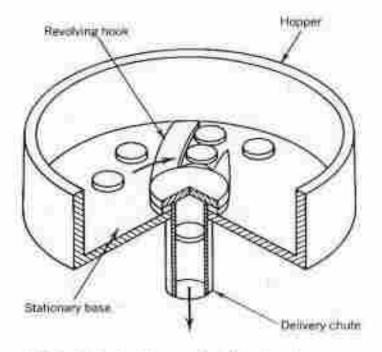


Figure (6.7). Revolving hook hopper for disk-shaped parts

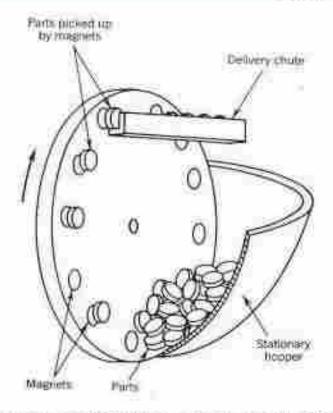


Figure (6.8a). Two types of magnetic feed hoppers for ferrous parts which are relatively flat, (a) magnetic disk feeder

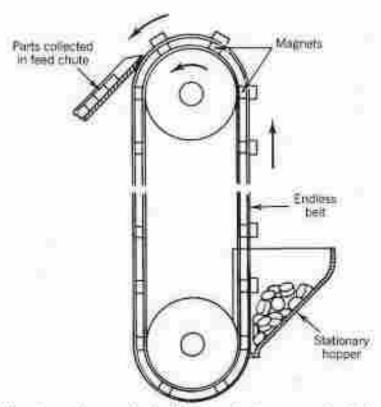


Figure (6.8b). Two types of magnetic feed hoppers for ferrous parts which are relatively flat, (b) magnetic elevating hopper feeder.

# 1.3. Orientation, Selection and Rejection

It can be seen that many of the transfer mechanisms that impart motion to the parts also orient them before discharging them onto a track or into a chute. However, the figures shown thus far have illustrated simple shapes such as cylinders and disks. The cylinders can be tapered or perhaps slightly compound, but for headed screws and other more complicated shapes a more sophisticated means of orientation is usually needed.

Returning to the vibratory bowl, the most versatile of feeding mechanisms, along the top of the bowl will be found some subtle irregularities in the inclined track that have been designed to recognize the geometry of the particular part being fed. These irregularities are designed to push, dump, or otherwise release incorrectly oriented parts to fall back down into the bowl for a retry at correct orientation. The headed screw or rivet is a good first example. Figure (6.9) shows several clever tricks to obtain the desired orientation of the rivets into the slot on the left, rivet head up. Taking advantage of the fact that the rivet is longer than the diameter of its head, a wiper blade diverts all rivets standing on their heads. Next, a pressure break causes the line of rivets to buckle if the delivery slot is too full to accept new rivets at the rate they are being pushed up from below. The pressure break also is a track-narrowing device that assures that the line of rivets will arrive single-file at the delivery slot. However, even at its narrowest point, the track accommodates the rivet turned either shank or head forward. Now note the slot, which is wide enough for the shank but not the head. The shank will fall into the slot whether the head or the shank is trailing. If the rivet is turned somewhat diagonally on the track so that the shank misses the slot, the downward slope on the inside shoulder of the slot causes the rivet to fall back into the bowl for a retry.

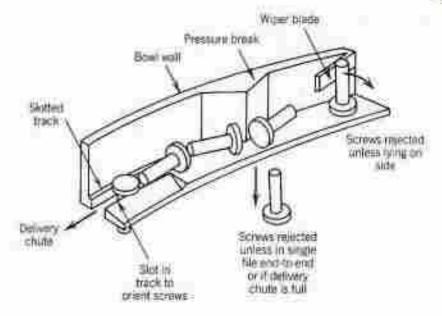


Figure (6.9). Orienting headed screws or rivets at the top discharge portion of a vibratory bowl

For washers or other disks, a device is needed to create a single file by dumping extras that may have climbed onto the backs of other washers. A sloped portion of the track with a tiny lip thinner than the washer does the trick, as shown in Figure (6.10).

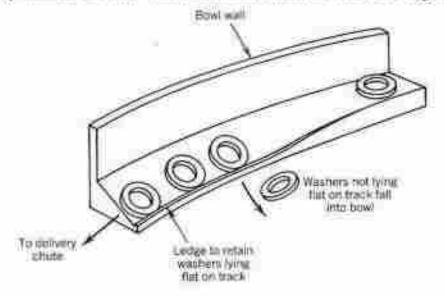


Figure (6.10). Creating a single file of washers in a vibratory feeder

For bottle caps or other cup-shaped objects, a wiper blade will work if the caps are shorter than their diameters (Figure (6.11)). Note that this rule is the opposite of the rule used earlier for screws and rivets. However, in the case of the caps the desired orientation happens to be open-end up. The scallop dumps any cap standing open-end down. Finally, a closed section of track can be used to invert the cap to a final useful orientation for capping oriented in the final, correct orientation, but some caps got that way too early in the process. The caps must be uniformly upside down when they reach the closed inversion track.

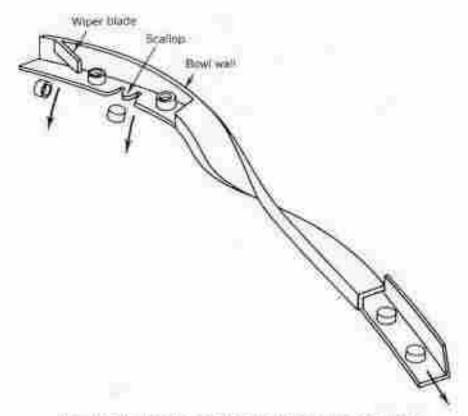


Figure (6.11). Feeding of bottle caps in correct orientation

The systems described thus far seem to apply primarily to small geometric shapes of metal. Such piece parts are typical, but the methods described here are not limited to the metal products industries. For the sake of variety, consider Figure (6.12), which illustrates a rotating drum for orienting plastic bottles. Consider the billions of plastic bottles consumed each year in the United States alone, and the motivation for automation comes into focus. In Figure (6.12), note that there are two chutes, one for bottles coming out cap end first and the other for bottom end first. One orientation is desired, but there is no reason why one line cannot be inverted and then merged with the other to produce a continuous line of correctly oriented bottles.

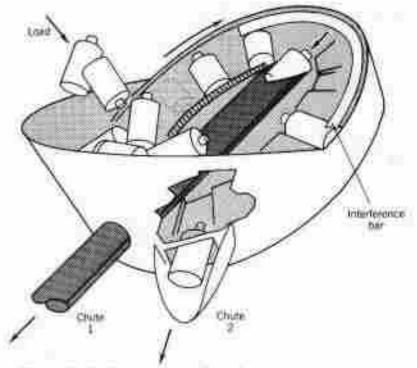


Figure (6.12). Rotating drum for orienting plastic bottles

## 2. PARTS FEEDING RESEARCH

There is more to parts feeding equipment design than creative genius. Some significant quantitative and experimental research is being conducted by both public and private institutions.

Studies shows that parts feeding design can be more scientific than most people think. Of course, changing the dimensions and center of mass of the part means that a new working range of slot widths must be identified. However, for standard shapes such as cylinders, disks, cubes, prisms, screws, and bolts of given materials, formulas can be derived that express slot width limits as a function of part geometries.

# 2.1. Selector Efficiency

Using probability theory, the efficiencies and outputs of various part selector combinations can be calculated if the transition probabilities of each selector phase are known. By "transition probability", we mean the chance that a piece part will change to orientation y when it passes through a selector device given that the part was in orientation x prior to encountering the device. The most popular mode of selector device

is the rejector, which simply discards an incorrectly oriented piece back into the center of the vibratory bowl or other hopper. The pure rejector is a device for which the transition probability from a given entry state to the rejected state is 100 percent. A device can be a pure rejector for some entry states and not others. It is possible to calculate directly the output orientation distribution for this combination of three devices for any given input distribution and also to calculate the system "efficiency", which we shall define as follows:

Efficiency = 
$$\frac{F_o}{F_I}$$
,

where Fo= output feed rate of correctly oriented parts;

 $F_I = total input feed rate.$ 

# 2.2. Efficiency versus Effectiveness

Another way to view the selector is in terms of "effectiveness" instead of efficiency. Selector effectiveness can be defined as follows:

Effectiveness = 
$$\frac{F_o}{F_r}$$
,

where  $F_T = total$  output feed rate.

Effectiveness is an interesting criterion for analysis, especially for individual selector stations. However, since most overall selector systems approach the 100 percent ideal for effectiveness, the more useful measure of the overall parts selector system is efficiency, not effectiveness.

# 2.3. Part Wear and Damage

A final consideration may be that the parts selector will overwork the parts, oscillating and vibrating them and kicking many of them back for retry after retry. If the effectiveness is virtually 100 percent, it is possible to use the system efficiency to calculate the chances that a part will be tossed back k times before reaching an acceptable orientation. The formula is:

$$p_k = \left[\frac{E}{100}\right]^1 \left[1 - \frac{E}{100}\right]^k$$

where  $p_k$  = probability that the part will be kicked back k times;

E = efficiency;

k = number of kickbacks.

Thus, nearly one out of a hundred parts will be kicked back ten times before achieving an acceptable orientation. The automation engineer must decide whether or not this kind of treatment will damage the product.

## AUTOMATIC PRODUCTION AND ASSEMBLY

After parts orientation and feeding, the process of preparing the parts for automatic processing or assembly. Production lines and assembly machines use a variety of pneumatic and electric drives and actuators. In the assembly cycle, many of the principles apply to other automatic processes such as transfer presses, gang drills, and automatic indexing lathes.

## 1. ASSEMBLY MACHINES

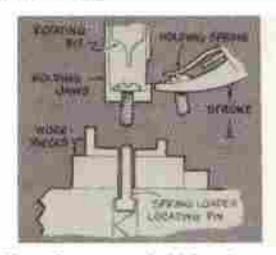
The term assembly machine usually implies multiple station operation with or without storage between stations. Some operations are common to most assemblies, however, and one of these, the driving of screws.

## 1.1. Automatic Screwdrivers

Screws are dumped in bulk into vibratory bowl hoppers where they are oriented and fed down a discharge track to the point at which a power-driven screwdriver is guided automatically into position to drive the screw (Figure (7.1)).

The mechanical jaws or "fingers" receive the screw from the feed track, hold it erect with the screw centerline aligned with that of the screwdriver, and then travel down with the screwdriver continuing to hold the screw until the automatic driver has engaged the slot and turned the screw several turns. Note the similarity between this operation and that of a human hand holding a screw to be driven manually.

The actual driver can be powered either electrically or pneumatically with advantages for each. Because the driver must disengage within a given torque range, a clutch or other method must be used to achieve this disengagement. Pneumatic motors may simply stall at approximately the correct torque if the air pressure is regulated correctly. If more air pressure is needed to achieve necessary spindle speeds to meet production requirements, or if electric drives are used, friction or ratchet clutches can be used. Even with quality components, automatic screw driving and other automatic assembly operations demand accurate fixturing to hold the parts properly during assembly.



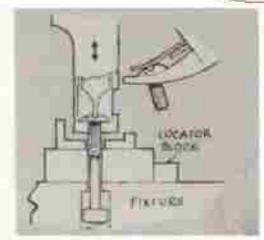


Figure (7.1). Automatic driving of screws. In (a) holding jaws receive the screw from the feeding and orientation system. In (b) the holding jaws move down with the tool until the screw enters the hole and contacts the locating pin

Nut and bolt assembly is quite similar to screw assembly operations. Bolts can be positioned with robots or linear actuators. Nuts can be attached using mechanisms similar to the automatic screwdriver pictured in Figure (7.1). Larger components and housings can usually benefit from fixtures unique to the assembly. Industrial robots with special grippers are useful for these components also.

# 1.2. Indexing Machines

The identity of the assembly machine is associated principally with the method of "indexing" or transferring the partially completed assembly from station to station.

The two principal types of indexing machines are in-line (Figure (7.2)) and rotary indexing. The archtype of the in-line style is the automobile assembly line, although automobile assembly lines retain a large number of manual operations. In-line arrangements are more flexible in that the method can accommodate any number of stations. Perhaps even more important is the convenience of adding in-process storage capability along the line. In-process storage capability is the key to increasing the productivity of lines that have a large degree of variability in individual station cycle times.

The rotary-indexing assembly machine is more popular for small parts assemblies.

These machines are sometimes called *dial indexing machines*. A circular table has several stations that hold the product in various stages of its assembly as it advances

around the circle. It is possible for the motion of the rotary table to be continuous, as in automatic bottling machines, but this requires die assembly apparatus to travel with the table. Most factory assembly machines advance in discrete intermittent steps. One station is used for the mounting of the main housing, base, or subassembly onto the rotary table. This step can be accomplished by a human operator or by an industrial robot. After the table makes nearly a complete circle, the completed assembly is removed at a station adjacent to the input station, or sometimes a single station serves both the loading and unloading functions.

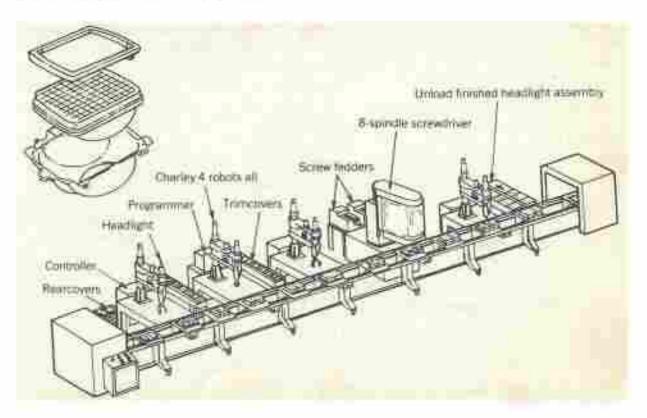


Figure (7.2).In-line, nonsynchronous, automatic transfer line for assembly of automobile headlamps

### 2. PRODUCTION AND THROUGHPUT

The automatic assembly machine, whether it is rotary or in-line, produces a completed assembly every time the machine indexes, regardless of the number of stations in the assembly process. To compute the (ideal) production rate of an automatic assembly machine, we need to know only the indexing cycle time; the number of stations is immaterial. It is true that throughput time, the time required to assemble a

given assembly from start to finish, is dependent upon the number of stations and the index time, but production rate is not.

# 2.1. Production and Throughput Time

# Example 1

An automatic assembly machine is of the dial-indexing configuration, has eight stations, and is driven by a geneva mechanism in which the driver has a rotational speed of 30 rpm. What is the production and throughput time of this machine?

# Solution

The principle of the geneva mechanism in which every revolution of the driver constitutes one indexing of the assembly machine. Therefore the production rate is thirty units per minute.

Production time = 
$$\frac{1}{\text{production rate}} = \frac{1}{30 \text{ units/min}} \times \frac{60 \text{ sec}}{\text{min}} = 2 \text{ sec per unit}$$

Throughput time = production time × number of stations

$$= 2 \sec \times 8 \text{ stations} = 16 \sec$$

We have been focusing upon ideal production rates in which the assembly machine functions without jamming, but this ideal is never completely achieved. Station malfunctions on an automatic assembly machine can have disastrous effects upon production, as will be seen in the section that follows.

# 2.2. Machine Jamming

It is easy to overlook the pitfalls of automation, and one of the most notorious examples can result from the setting up of automatic multistation assembly machines without due consideration to the potential effects of station malfunction or jamming.

Consider an eight-station rotary indexing machine driven by a geneval mechanism in which the index time is three seconds and the dwell time is five seconds. Checking the validity of the ratio of index to dwell time. Under ideal operating conditions (no malfunctions), this eight-station rotary indexing machine will produce a completed assembly every eight seconds and will achieve a corresponding **production rate of 450 units per hour**.

In a more realistic case, suppose each station malfunctions on the average of once every 100 cycles, a seemingly tolerable rate of work stoppage. One or more station malfunctions will immediately jam the indexing machine, requiring an operator to make adjustments to restart the machine. For our example, let us say that this adjustment and restart process requires a mere ten minutes. The result is a reduction in productivity of the automated assembly machine, as can be seen in the following series of calculations.

If the chance of a station malfunction is one in 100, the chance that a given station will <u>not</u> malfunction in a given cycle is 99 % or 0.99. But all eight stations must operate without malfunction to produce a completed assembly successfully.

So the probability of no malfunction at each station during that cycle. Multiplying the station success probabilities:

$$0.99^8 = 0.9227$$

Therefore, out of 10,000 machine cycles, 9227 assemblies will be produced without malfunction, with a cycle time of eight seconds per assembly. This will consume:

In the other 773 cycles (10,000-9227), at least one station will malfunction and require a ten-minute repair. This will consume:

The total time to produce the 9227 assemblies is then:

For the total of operating time plus malfunction downtime. The percent downtime is:

$$\frac{128.83}{149.33} = 0.863 = 86.3 \%$$
 of the total production time.

The production rate has been reduced from the ideal of 450 units per hour calculated earlier to:

$$\frac{9227 \text{ units}}{149.33 \text{ hr}} = 61.8 \text{ units/hr}$$

The efficiency of the assembly machine would be the ratio of the actual production rate to the ideal production rate, calculated as:

Efficiency = 
$$\frac{61.8 \text{ units/hr}}{450 \text{ units/hr}} = 0.137 = 13.7 \%$$

an amazing 86.3 % drop in efficiency from introduction of only a slight (1 %) station malfunction rate. Such is the world of automation, and the automation engineer should be prepared to deal with the realities of system malfunction and downtime when planning for a new automated assembly machine.

# 2.3. Component Quality Control

Explaining why these malfunctions might occur. The predominant cause of assembly station malfunction is some random variation in the components being assembled, variation of a magnitude that cannot be handled by the assembly machine. If tighter specifications can be applied or closer quality control can be exacted upon the components produced to existing specifications, the automation engineer may be able to achieve astonishing improvements in assembly machine production rates. Suppose in the example in the previous section that 90 % of the assembly malfunctions were due to faulty components. Elimination of the component quality problem would then reduce the station malfunction rate from one out of 100 cycles to one out of 1000 cycles. Let us now refigure the production rate and percent downtime with the component quality problem eliminated.

Probability of at least one station malfunction in a given typical cycle:

$$0.999^8 = 0.9920$$

So in 10,000 machine cycles, 9920 assemblies will be produced with a cycle time of eight seconds per completed assembly. This will consume:

In the other 80 cycles (10,000 - 9920), at least one station will malfunction and require a ten-minute repair time. This will consume:

Now the total time to produce 9920 assemblies is:

$$22.04 \text{ hr} + 13.33 \text{ hr} = 35.37 \text{ hr}$$

for both operating time and downtime. But the percent downtime has been reduced from 86.3 % to:

of the total production time. The production rate is back up to:

which is:

$$280.46/450 = 0.62 = 62 \%$$
 of the ideal production rate.

A production efficiency of 62 % is not very good either, but it is considerably better than the 13.7 % achieved without component quality control. It can be seen that the control of component quality can easily become the difference between success and failure of an automated assembly system setup. A case study will now serve to review the essential concepts of the previous two sections.

# Example 2

# (Automatic Assembly with Station Malfunctions and Parts Jamming)

An in-line automatic transfer and assembly machine has thirty consecutive assembly stations. The line is under control of a walking beam with an index time of four seconds and a dwell of twenty seconds. Each station along the line will operate without malfunction with a reliability of 0.999 when its hopper is supplied with quality components. Any defective component will cause a station to jam, which in turn will precipitate a line jam because there is no provision for in-process assembly storage along the line. A jam or malfunction requires ten minutes to correct.

# a) No station malfunction or jamming

Assuming no station jamming or malfunction at all, what is the ideal production capability of this line? What is the throughput time?

### Solution

Cycle time

$$T_c = 4 \text{ sec index} + 20 \text{ sec dwell} = 24 \text{ sec}$$

Ideal production rate

$$R = \frac{3600 \text{ sec/hr}}{24 \text{ sec/unit}} = 150 \text{ units/hr}$$

Throughput time

$$T_T = 30 \text{ stations} \times 24 \text{ sec/station} = 720 \text{ sec} = 12 \text{ min}$$

## b) Assuming station malfunction

Assuming ideal component quality, what is the production rate considering station malfunction? What is the percent downtime? What is the throughput time?

## Solution

Prob [a cycle will not malfunction] = 
$$0.999^{30} = 0.9704$$

Prob [a cycle will malfunction] = 
$$1 - 0.9704 = 0.0296$$

Time to produce 9704 assemblies

Total time = total successful cycle time + malfunction correction time

= 9704 units × 24 
$$\frac{\text{sec}}{\text{unit}}$$
 ×  $\frac{1 \text{ hr}}{3600 \text{ sec}}$   
+ 296 malfunctions × 10  $\frac{\text{min}}{\text{malfunction}}$  ×  $\frac{1 \text{ hr}}{60 \text{ min}}$   
= 64,69 hr + 49,33 hr = 114,02 hr

Production rate

$$R = 9704 \text{ units}/114.02 \text{ hr} = 85 \text{ units/hr}$$

Percent downtime

$$D = 49.33 \text{ hr}/114.02 \text{ hr} = 0.43 = 43 \%$$

Throughput time

$$T_T = 30 \ stations \times \frac{1}{85 \ units/hr} \times 60 \ \frac{min}{hr} = 21.18 \ min.$$

## c) Assuming defective parts jamming

Assuming component lot quality is at the level of (1/2) of one percent defective, what is the effect of defective components upon production rate?

### Solution

Time to produce 8349 assemblies

Total time = total successful cycle time + unjam time

= 8349 units × 24 
$$\frac{\text{sec}}{\text{unit}}$$
 ×  $\frac{1 \text{ hr}}{3600 \text{ sec}}$   
+ 1651 jams × 10  $\frac{\text{min}}{\text{jam}}$  ×  $\frac{1 \text{ hr}}{60 \text{ min}}$   
= 55.66 hr + 275.16 hr = 330.82 hr

Production rate

$$R = 8349 \text{ units/}330.82 \text{ hr} = 25.24 \text{ units/}hr$$

Percent downtime

$$D = 275.16 \ hr/330.82 \ hr = 0.83 = 83 \%$$

Efficiency

$$E = \frac{\text{actual production rate}}{\text{ideal production rate}} = \frac{25.24 \text{ units/hr}}{150 \text{ units/hr}} = 0.17 = 17 \%$$

Throughput time

$$T_T = 30 \ stations \times \frac{1}{25.24 \ units/hr} \times 60 \ \frac{min}{hr} = 71.32 \ min = 1.19 \ hr$$

Thus, a component quality level of only (1/2) of one percent defectives in this case reduces the production rate from 95 units/hr to 25.24 units/hr, increases downtime from 43 % to 83 %, and throughput time is increased from a little under twenty minutes to over an hour.

So important is the quality and uniformity of components to the success of assembly automation that some firms pay several times the standard price for such items as screws, nuts, and bolts to purchase the finest quality available for use in automated assembly. The prohibitive cost of machine jamming due to faulty components easily justifies the added cost for quality components in these cases.

# 24. Defective Component Assembly

One way to deal with the component quality problem is to use automatic in-process inspection to prevent costly assembly operations from being wasted on defective components. This is a good strategy, but the results of Example 2 suggest that once the components reach the assembly machine a better strategy may be to assemble the defective components intentionally rather than stop the line. For some assemblies electronic circuit boards, for instance it may be more practical to screen out defective components after the assembly is complete, even though this entails the added cost of partial disassembly. The benefits of this strategy will be illustrated in Example 3.

### Example 3

### (Savings from Defective Component Assembly)

A twenty-station automatic assembly line operates on a two-shift basis, including maintenance, 350 days per year. Preventive maintenance is performed on the third shift and during normal operation. The line is stopped upon demand for machine repair or defective component replacement. The component quality level is one-percent defective, and the current strategy is always to detect and remove defective components before assembly, even though such removal requires stopping the line an average of fifteen minutes each time to remove defective components. The

automatic assembly machine has an equivalent annual cost of \$490,000 and achieves an ideal cycle time of ten seconds.

A strategy is proposed to keep the line running without stopping to remove defective components, even though this will entail rework at a cost of 0.6 labor hours at \$20 per hour per defective assembly. Is the savings in production time worth the expensive rework?

### Solution

The analysis of this strategy consists of comparing the typical cost of line downtime (alternatives):

Cost A = 15 min/stoppage 
$$\times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{16 \text{ hr}} \times \frac{\$490,000/\text{yr}}{350 \text{ days}} = \$21.875$$

With the typical rework cost (alternative B):

$$Cost B = 0.6 \ hr/rework \times $20 \ hr = $12$$

The better alternative of the two is to assemble defective components intentionally.

Note in Example 3 that production rate and cycle time did not enter the analysis. The lower cost alternative in this case study was to assemble defective components and rework later, but other alternatives should be considered. The assembly machine downtime for component removal is not really lost if it can be made up on the third shift. The equivalent annual cost of the machine itself would remain essentially the same since most of this cost would consist of capital recovery of the original investment in the machine. Two other alternatives would be:

- ✓ automatic preinspection screening of the components to remove defectives before they reach the assembly machine;
- ✓ improvement of the source quality of the components during their manufacture.

Example 3 was a concoction of circumstances that would make the intentional automatic assembly of defective components a viable alternative. Usually, however, this alternative will not be the best one. If there is any idle capacity on the machine, even on the night shift, it will usually be advantageous to use that idle capacity to make up for

downtime rather than incurring rework costs due to intentional assembly of defective component

In case of considering the circumstances of Example 3 to be too farfetched, it should be mentioned that even in conventional manufacture the intentional acceptance of known defectives is a calculated strategy. For instance, in the garment industry, fabric flaws are often detected by the cutting-room personnel during the spreading of lays before the cut is made in thicknesses of up to 250 ply. The flaws are left in the fabric to avoid disrupting the laying process and to prevent damage to other pieces that will be cut from the defective ply. It is usually hoped that later the defective pieces will be discovered and removed, either in the sewing room prior to stitching or even in final inspection where rework will be the unfortunate consequence. Either alternative may be preferable to removing the flaw in the cutting room.

# 2.5. Buffer Storage

If a large degree of random variation exists in the operation of the individual stations along an automatic assembly line, a degree of independence between the stations must be built in to achieve even a modicum of efficiency. This independence is achieved by placing buffer storage between stations. Such a system usually dictates a nonsynchronous material transfer system because a rigid, intermittent transfer system does not accommodate the variations in station operation time.

To see the advantage of buffer storage between stations, consider a line in which the ideal cycle time is one minute. If each station on the line malfunctions once every sixty cycles and one hour is required to resolve each malfunction, each station will be available for production approximately 50 % of the time. Hypothetically, if all of the station malfunctions could be synchronized, the line could theoretically achieve a production of one-half of sixty, or thirty, units per hour. But the nature of malfunctions is that they occur at random points in time. This characteristic greatly reduces the productivity of the line, because if any station along the line is down, the entire line is down as was seen earlier in the analysis of

indexing assembly machines. In the situation described above in which each station is available only 50 % of the time, the availability of the entire indexing line is 0.5<sup>n</sup>, where n represents the number of stations along the line. Such a line with only five stations would be down 97 % of the time! The difference between this percentage and the hypothetical percentage of availability if the malfunctions could be synchronized:

$$97\% - 50\% = 47\%$$

represents the production loss due to interference between the stations. The interference is called "blocking" when station (i) cannot release its part to station (i + I) and is called "starving" when station (i) cannot obtain a part from station (i-I). In either case, station i is idle during this period even if it is not malfunctioning and would be able to produce if it had a part upon which to perform its operation.

The provision of buffer storage between stations permits each station to produce "to stock." The first station is started first and produces approximately one-half the capacity of its storage before the second station is started. The second station then begins, working from the stored supply produced by the first station while the first station continues to produce. Subsequent stations are added to production as each sequential storage reaches approximately one-half its capacity, provided stations are reasonably similar in operating characteristics and downtime. When all storages are thus loaded, the line can function with some resilience due to the buffer storages between stations. If the buffer storages are large enough to prevent blocking and starving, the production of the line described earlier can be back up to thirty units per hour.

## **Tutorial Sheet No 2**

## Q: 1

## Buffered assembly line production

Example 2 (lecture No.7) considered the devastating effects of station malfunctions and parts jamming upon automatic assembly. Suppose the same thirty-station operation were performed on a buffered assembly line with the same twenty-second productive cycle. (Note that the four-second index time is ignored in this case because each buffered station will be producing to stock). Calculate the improvement in line productivity.

#### Sol.

Prob [a station will not jam] = 0.994 (from solution to Example 2 (c))

## Time to produce 994 assemblies

### Production time

$$= 994 \text{ units} \times \frac{20 \text{ sec}}{\text{unit}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} + 6 \text{ jams}$$

$$\times 10 \frac{\text{min}}{\text{jam}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 5.52 \text{ hr} + 1 \text{ hr} = 6.52 \text{ hr}$$

## Production rate (buffered)

$$R_B = 994 \text{ units/6.52 hr} = 152.4 \text{ units/hr}$$

## Production rate (unbuffered)

## Productivity improvement

$$\frac{R_B}{R} = \frac{152.4}{25.24} = 6.04$$

Thus, Q: 1 shows\_how the addition of adequate buffer storage could increase the productivity of the line six times. In fact, fully buffered assembly lines are really a degeneration to batch-type manufacturing, typical of a plant prior to automation.

Q: 1 omitted a calculation of throughput time which goes up when buffer storages are provided. The amount of the increase is depending upon the amount of buffer storage provided, as will be illustrated in the next question (Q: 2).

## Q: 2

### Effect of Buffer Storage upon Throughput Time

A thirty-station assembly line with buffer storage between stations that averages fifty units each has a production rate of 152.4 units per hour (the same as in Q: 1). Calculate throughput time.

#### Sol.

Throughput time must consider the wait time each unit spends in the buffer storage between stations. The average wait time is the product of the average station cycle time (including downtime) times the average storage number in storage. Thus, for an average storage quantity of fifty:

Average wait time (between any two stations) = 
$$\frac{1}{152.4 \text{ units/hr}} \times 50 = 0.328 \text{ hr/buffer}$$

There would be n-1 or twenty nine buffer storages in a thirty-station buffered assembly line would be:

Total average wait time = (average wait time in each buffer) × number of buffers = 0.328 hr/buffer × 29 buffers = 9.51 hr

Total throughput time = production time + wait time

$$= \left[ \frac{1}{152.4 \text{ units/hr}} \times 30 \text{ stations} \right] + 9.51 \text{ hr}$$
  
= 0.20 hr + 9.51 hr = 9.71 hr

#### Another Solution

Another way to calculate throughput time for the buffered assembly line is to recognize that there are

$$30 + (29 \times 50) = 1480$$

positions throut which a unit must advance through the system at a rate of 1/152.4 house per position:

Throughput time = 
$$[30 + (29 \times 50)] \frac{1}{152.4} = 9.71 \text{ hr}$$

The throughput time of 9.71 hours calculated for **Q**: **1** should be compared with the throughput time calculated for *Example 2(lecture No.7)*, part (c), which considered station malfunctions and jamming. That throughput time was calculated to be approximately one hour. Thus, although adding buffer storage increased line productivity six times (*Example 3*), it increased throughput time almost ten times (**Q**: **1**). This conclusion is based upon the assumption that an average buffer storage of fifty units would be ample to prevent any blocking or starving of stations in the buffered assembly line.

# 8.1 Distinguishing Between NC, DNC, and CNC

- Numerical Control NC is a form of programmable automation in which the mechanical actions of a machine tool or other equipment are controlled by a program containing coded alphanumerical data.
- Numerical control NC is any machining process in which the operations are executed automatically in sequences as specified by the program that contains the information for the tool movements.
- The alphanumerical data represent relative positions between a workhead and a
  workpart as well as other instructions needed to operate the machine.
- The workhead is a cutting tool or other processing apparatus, and the workpart is the object being processed.
- A CNC unit of a machine tool consists of one ormore central processing units (CPUs), input/output devices, operator interface devices, and programmable logical controllers.
- The CNC unit may contain several CPUs, or microprocessors, depending on the
  tasks required by the machine tool. A very basic three-axis CNC milling machine
  requires the fine coordinated feeding velocity and position control of all three axes
  and the spindle speed simultaneously. Current CNC systems tend to use multiple
  CPUs depending on the number of computation tasks. CNC systems, equipped
  with a monitor and a keyboard, allow operators to edit NC programs on site.
- NC programs, production schedules, and recording of production times and operation cycles are continuously electronically communicated between the CNC units and the master computer. Such systems are called Distributed Numerically Controlled (DNC) systems.

### 8.2 Introduction to CNC Machine

A diagram of a typical three-axis computer numerically controlled (CNC) machining center is shown in Figure 6.1. The CNC machining center consists of mechanical, power electronic, and CNC units. The mechanical unit consists of

beds, columns, spindle assembly, and feed drive mechanisms. Spindle and feed drive motors and their servoamplifiers, high-voltage power supply unit, and limit switches are part of the power electronics group. The CNC consists of a computer unit and position and velocity sensors for each drive mechanism. The operator enters the numerically controlled (NC) program to the CNC unit. The CNC computer processes the data and generates discrete numerical position commands for each feed drive and velocity command for the spindle drive. The numerical commands are converted into signal voltage (±5V or ±10 V) and sent to servoamplifiers of analog drives, or sent numerically to digital drives that process and amplify them to the high-voltage levels required by the motors. As the drives move, sensors measure their velocity and position. The CNC periodically executes digital control laws at fixed sampling intervals that maintain the feed speed and tool path at programmed rates by using sensor feedback measurements.

The fundamental principles of designing CNC systems are covered in this chapter. First, the sizing and selection of drive motors are presented, followed by physical structure and modeling of a servodrive control system. The mathematical modeling and analysis of drive systems are covered both in the time and frequency domain.

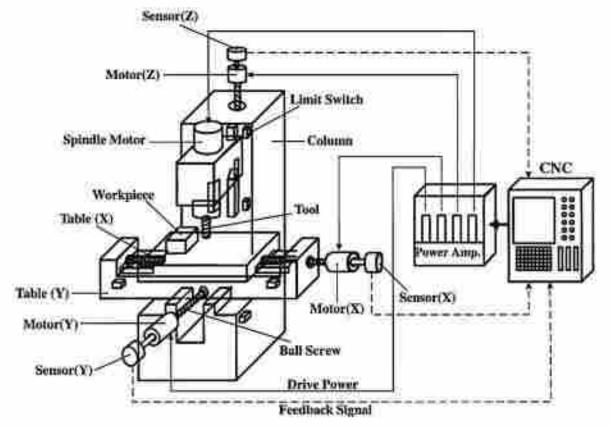


Figure 6.1: Functional diagram of a three-axis CNC machining center.

### 8.3 Machine Tool Drives

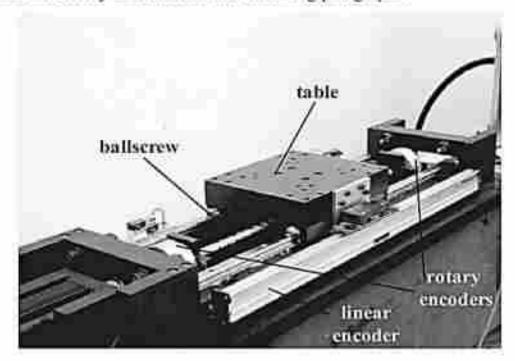
The drives in machine tools are classified as spindle and feed drive mechanisms. Spindle drives rotate over a wide velocity range (i.e., up to 35,000 rev/min), whereas the feed drives usually convert angular motions of the motors to linear traverse speeds, which can range up to 30,000 mm/min. In this text, the servocontrol of feed drives is covered only, although the material can be easily extended to spindle drives because the fundamental design and analysis methods are quite similar.

Let us take one of the feed drives in a machine tool as an example. The feed drive has the following mechanical components: the table with a workpiece, the nut, the ball lead screw, the torque reduction gear set, and the servomotor (see Fig. 6.2). Because of their efficient torque delivery capability at various speeds, themost common servomotors used in the feed drives are direct current (dc) motors. However, alternating current (ac) servomotors have also gained popularity because of their improved performance. The electrical components of a servomotor system comprise the servomotor amplifier, velocity and position feedback transducers, a digital computer, and a digital to analog converter circuit.

# 8.3.1 Mechanical Components and Torque Requirements

The feed drive motor has to overcome both the static and dynamic loads in the machine tool. The sources of the static loads are the friction losses in the guideways and bearings and the cutting forces acting in the feeding direction of the table. The motor must deliver a high enough dynamic torque to accelerate the table, workpiece, and leadscrew assembly for a short period of time until the drive reaches the desired steady-state speed. The dynamic torque is given as peak torque or peak current delivery with a period of 2 to 3 seconds by the servomotor manufacturers. The motors must have a sufficiently high continuous torque delivery range and a sufficient peak torque and delivery period to overcome the static and dynamic loads, respectively. Estimation of static and dynamic motor

loads are briefly introduced in the following paragraphs.



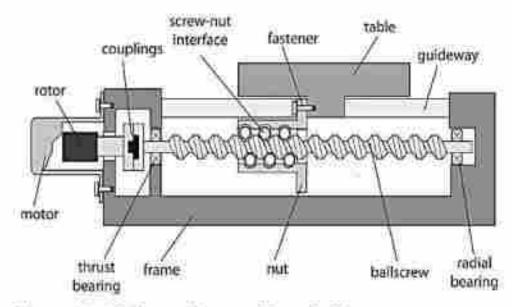


Figure 6.2: A ballscrew-driven machine tool table.

### Static Loads

There are three sources of static loads: the friction in the slideways, frictional losses in the feed drive bearings, and cutting forces. The friction in the guideways depends on the type of contact between the sliding table and the stationary guideway. High friction coefficients are found at plain lubricated guideways where the metal-to-metal table-guideway surface contact area is large. The metal contact surface area

is reduced in hydrostatic and hydrodynamic guideways where pressurized lubricant is injected between the table and the guideway. The friction coefficient is probably smallest in the guideway designs where roller bearings are used at the guidewaytable assembly (see Fig. 6.2). The torque reflected on the feed drive motor due to friction  $(T_{ef})$  in the guideways can be estimated as

$$T_{gf} = \frac{h_g}{2\pi} \mu_{gf} [(m_t + m_w)g + F_s],$$
 (6.1)

where  $\mu_{gf}$  is the friction coefficient on the guideways,  $m_t$  is the table mass,  $m_w$  is the workpiece mass,  $F_z$  is the normal cutting force on the table,  $h_p$  is the leadscrew pitch length, and g is the gravitational acceleration (9.81 m/s<sup>2</sup>).

The friction coefficient for plain guideways typically ranges from 0.05 to 0.1 and the vertical cutting force  $(F_x)$  can be taken as 10 percent of the maximum resultant cutting force in a typical vertical milling machine tool [105].

Axial thrust bearings are used at both ends of the leadscrew to absorb the feed forces and also to guide the screw radially at the same time [109]. Axial thrust bearings are preloaded in tension to offset the backlash produced by the thermal expansion of the leadscrew because of friction in the feed drive assembly. In addition to the preload, the axial thrust bearings are loaded by the feed forces. The feed forces can be estimated by using the cutting mechanics relationships given in the metal cutting chapter. The torque lost in the bearings and preload is estimated as

$$T_{lf} = \mu_b \frac{d_b}{2} (F_f + F_p),$$
 (6.2)

where  $\mu_b$  is the friction coefficient on bearings (typically approximately 0.005),  $d_b$  is the mean bearing diameter,  $F_t$  is the maximum feed force on the table, and  $F_b$  is the preload force.

The torque reflected on the leadscrew shaft due to the cutting forces in the feed direction is given as

$$T_f = \frac{h_g}{2\pi} F_f. \qquad (6.3)$$

The total static disturbance load reflected on the leadscrew shaft  $(T_s)$  is found by summing the three torque values calculated in Eqs. (6.1), (6.2), and (6.3) as follows:

$$T_e = T_{ef} + T_{ff} + T_f$$
, (6.4)

In the cases where the static torque  $(T_s)$  is too large, a gear reduction can be applied between the motor shaft and the leadscrew. The gear reduction ratio  $(r_s)$  is defined as

$$r_{t} = \frac{z_{t}}{z_{m}} = \frac{n_{m}}{n_{t}},$$
 (6.5)

where  $z_m$  is the number of teeth on the motor's gear,  $z_l$  is the number of teeth on the feedscrew's gear,  $n_m$  is the motor's angular velocity (rev/min), and  $n_l$  is the feedscrew's angular velocity (rev/min). To reduce the speed, we must have  $z_l > z_m$ , which gives a gear reduction ratio larger than one (i.e.,  $r_{\rm ff} > 1$ ). The reduced reflected torque on the motor's shaft  $(T_{\rm rr})$  is found as

$$T_{sr} = \frac{T_s}{r_c}.$$
(6.6)

The CNC designer must select a dc motor that has a larger continuous torque delivery capacity than the static torque reflected on the motor's shaft.

### **Dynamic Loads**

Machine tools require high-acceleration torque during speed changes. The reflected inertia on the motor's shaft consists of the inertia of the table, the workpiece, the leadscrew, the gears, and the motor's shaft. The moment of inertia of the table and workpiece reflected on the leadscrew shaft is

$$J_{\text{tw}} = (m_1 + m_w) \left(\frac{h_p}{2\pi}\right)^2$$
. (6.7)

The moment of inertia of the leadscrew with a pitch diameter of  $d_p$  is

$$J_l = \frac{1}{2} m_l \left(\frac{d_p}{2}\right)^2, \qquad (6.8)$$

where  $m_l$  is the mass of the leadscrew shaft. The total inertia reflected on the motor's shaft is

$$J_u = \frac{J_{tw} + J_l}{r_{tl}^2} + J_m,$$
 (6.9)

where  $J_m$  is the inertia of the motor's shaft, and  $r_g \ge 1$  is the gear reduction ratio between the feedscrew and motor speeds.

There is another friction torque in the drive system that is proportional to the velocity, namely, the viscous friction torque. The total dynamic torque required to accelerate the inertia  $J_e$  and to overcome viscous friction and the static loads is given as

$$T_{\rm d} = J_{\rm e} \frac{d\omega}{dt} + B\omega + T_{\rm ef}, \qquad (6.10)$$

where  $\omega$  is the angular velocity of the motor and B is the viscous friction coefficient. Note that, because the cutting is performed at low feeds, the contribution of cutting forces to the static torque  $(T_s)$  does not have to be considered in Eq. (6.10). The peak torque value delivered by the motor must be larger than the dynamic torque calculated from Eq. (6.10). If a gear reduction is used between the motor's shaft and leadscrew, the dynamic torque reflected on the motor's shaft is reduced; see Eqs. (6.9) and (6.10).

Example. A vertical milling machine is to be retrofitted with three identical dc servomotors. Because the largest load is applied in the longitudinal axis, the motors are selected according to the torque requirements of this axis. The

following parameters are given for the feed drive axis:

 $m_{\rm t} = 20 \, \rm kg - table mass,$ 

 $m_{\psi} = 30 \text{ kg} - \text{maximum mass for the workpiece.}$ 

 $m_l = 2 \text{ kg} - \text{leadscrew mass},$ 

 $h_p = 0.020 \text{ m/rev.} - \text{pitch of the feedscrew,}$ 

 $d_p = 0.020 \text{ m} - \text{feedscrew diameter},$ 

 $J_m = 2.875 \times 10^{-4} \text{ kg m}^2 - \text{motor's shaft, coupling, encoder}$ and tachonerator inertia.

 $r_{\rm g} = 1$ . – gear reduction ratio,

 $\mu_{\rm eff} = 0.1$  - friction coefficient in the guides,

 $\mu_b = 0.005$  - friction coefficient of bearings,

 $F_{\epsilon} = 1,000 \text{ N} - \text{maximum vertical force},$ 

B = 0.005 Nm/(rad/s) - viscous damping coefficient

 $F_f = 5,000 \text{ N} - \text{maximum feeding force},$ 

 $F_p = 2,000 \text{ N} - \text{preload force in thrust bearings},$ 

 $a_t = 5 \text{ m/s}^2 - \text{desired}$  acceleration of the table.

### Static Torque

The static torque contributed by the friction in the guideways (Eq. 6.1) is

$$T_{\text{gf}} = 0.1 \frac{0.020}{2\pi} [(20 + 30)9.81 + 1,000] = 0.4744 \text{ Nm}.$$

The torque lost in the bearings due to friction (Eq. 6.2) is

$$T_{\rm if} = 0.005 \frac{0.02}{2} (5,000 + 2,000) = 0.3500 \,\text{Nm}.$$

The torque required to overcome feed forces (Eq. 6.3) is

$$T_t = \frac{0.020}{2\pi}$$
5,000 = 15.90 Nm.

Thus, the total required continuous torque from the dc motor (Eq. 6.4) is

$$T_{st} = \frac{0.4744 + 0.35 + 15.90}{1.0} = 16.72 \,\text{Nm}.$$

# Dynamic Load

The moment of inertia of the table and workpiece reflected on the feedscrew shaft (Eq. 6.7) is

$$J_{\text{tw}} = (20 + 30) \left( \frac{0.020}{2\pi} \right)^2 = 5.066 \cdot 10^{-4} \text{ kg m}^2$$
.

The leadscrew's inertia (Eq. 6.8) is

$$J_l = \frac{1}{2}2.0\left(\frac{0.02}{2}\right)^2 = 1 \cdot 10^{-4} \text{ kg m}^2.$$

Because the motor is directly connected to the leadscrew of the machine (i.e.,  $r_g = 1.0$ ), the total inertia reflected on the motor's shaft is found (Eq. 6.9) to be

$$J_0 = 5.066 \cdot 10^{-4} + 1 \cdot 10^{-4} + 2.875 \cdot 10^{-3} = 8.9411 \cdot 10^{-4} \text{ kg m}^2$$
.

The angular acceleration of the motor's shaft is

$$\frac{d\omega}{dt} = \frac{a_I}{(h_u/2\pi)} = \frac{5}{0.020} 2\pi = 1,570 \text{ rad/s}^2.$$

The dynamic torque required is found from Eq. (6.10) as follows:

$$T_{\rm d} = 8.9411 \cdot 10^{-4} \, {\rm kg \ m^2} \times 1.570 \, {\rm rad/s^2} + 0.005 \, {\rm Nm/(rad/s)}$$

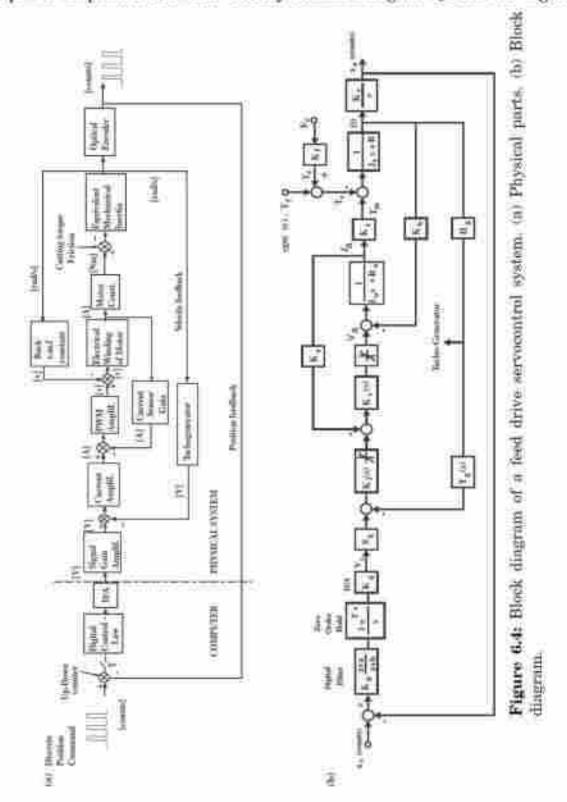
$$\times \frac{0.5}{0.020} 2 \pi ({\rm rad/s}) + 16.72 \, {\rm Nm} = 18.90 \, {\rm Nm}.$$

Thus the selected servomotor must be able to attain 18,90 Nm dynamic torque for a period of acceleration (0.1 s).

## dc Motor Power Amplifiers and Velocity Control Loop

The armature voltage is supplied by a power amplifier. The power amplifier receives a large constant dc voltage from a transformer, which converts ac line voltage to the desired dc voltage level. The power amplifier illustrated in this text is a pulse width modulated (PWM), current-controlled amplifier. However, the operation of other types of power amplifiers are quite similar.

The block diagram of a complete velocity control loop is shown in Figure 6.4. The power amplifier receives a velocity command signal  $V_\varepsilon$  from the digital to



analog converter output of the digital controller. The velocity command signal is first buffered with an adjustable differential preamplifier represented by a gain  $S_{\theta}$ . The output of  $S_{\theta}$  is compared with an actual velocity signal measured by the tachometer feedback unit. The resulting velocity error signal (in volts) is converted to a demand current by the current amplifier, which has a gain of K<sub>I</sub>. Most current amplifiers use an armature current feedback signal to improve the dynamic response of the motor. The feedback current signal is pulled from a current sense coupler and compared with the demand current. The PWM circuit generates a varying sawtooth shape dc voltage at certain frequency. The PWM frequency is usually higher than 10 kHz, Low-frequency PWM switching signals (up to 6 kHz) generate irritating audible noise. The current error signal is modulated by the PWM circuit, which is modeled as a gain K<sub>c</sub>. The resulting dc voltage becomes an ON-OFF type rectangular waveform. The average voltage level of the waveform (dc value) is used as armature voltage Va in the calculations. The complete block diagram of the amplifier, motor, and tachometer feedback unit are shown in Figure 6.4a.

The block diagram is organized with temporary state variables to illustrate the derivation of the velocity loop transfer function. The following relationships can be expressed from the block diagram by the use of temporary states,  $V_1$ ,  $V_2$ , and  $V_3$  as follows:

$$\begin{split} V_1(s) &= S_g V_c(s) - T_g H_g \omega(s), \\ V_2(s) &= K_1 V_1(s) - K_a I_a(s) \\ &= K_1 S_g V_c(s) - K_1 T_g H_g \omega(s) - K_a I_a(s), \\ V_3(s) &= K_v V_2(s) - K_b \omega(s) \\ &= K_v K_1 S_g V_c(s) - (K_v K_1 T_g H_g + K_b) \omega(s) - K_v K_a I_a(s). \end{split}$$

The transfer function between the current and state  $V_{\alpha}$  is

$$I_n(s) = \frac{V_0(s)}{L_n s + R_n}.$$

Substituting the value of  $V_3$  into the current expression yields

$$I_a(s) = \frac{K_v K_1 S_g}{L_s s + R_s + K_v K_s} V_e(s) - \frac{K_v K_1 T_g H_g + K_b}{L_u s + R_u + K_v K_s} \omega(s).$$
 (6.17)

The motor's mechanical transfer function (see Eq. 6.15) is

$$\omega(s) = \frac{T_m(s) - T_s(s)}{J_{\alpha s} + B}$$

œ

$$\omega(s) = \frac{K_t}{J_e s + B} I_u(s) - \frac{1}{J_e s + B} T_u(s).$$
 (6.18)

Substituting the current expression (6.17) into Eq. (6.18) yields the transfer function between the output velocity  $\omega$  and the velocity command input voltage

 $V_{\epsilon}$  and the disturbance torque  $T_{\epsilon}$  as follows:

$$\omega(s) = \frac{K_1}{s^2 + K_2 s + K_3} V_e(s) - \frac{(1/J_*)[s + (R_n + K_v K_a)/L_a]}{s^2 + K_2 s + K_3} T_e(s), \quad (6.19)$$

where

$$\begin{split} K_{1} &= \frac{K_{t}S_{g}K_{1}K_{v}}{L_{a}J_{e}}, \\ K_{2} &= \frac{B}{J_{E}} + \frac{R_{a} + K_{v}K_{a}}{L_{s}}, \\ K_{3} &= \frac{B\left(R_{a} + K_{v}K_{a}\right) + K_{t}\left(K_{b} + H_{g}T_{E}K_{v}K_{1}\right)}{J_{e}L_{a}}. \end{split}$$

The feed drive servovelocity controller is designed to have a fast rise time with zero overshoot at step changes in the velocity. Let us analyze the velocity loop as a function of the velocity command input voltage  $V_c$ . The transfer function (Eq. 6.19) can be expressed as

$$\frac{\omega(s)}{V_c(s)} = \frac{K_1}{s^2 + 2 \xi \omega_0 s + \omega_0^2}.$$
(6.20)

Here, the natural frequency  $(w_n)$  and damping ratio  $(\xi)$  of the velocity loop are defined as follows:

$$\omega_n = \sqrt{K_3}$$
 [rad/s],  
 $\xi = \frac{K_2}{2\sqrt{K_3}} < 1$ , (6.21)

where  $K_1, K_2 > 0$ . The time domain step response of this underdamped velocity servo is expressed as

$$\omega(t) = V_c \cdot \frac{K_1}{K_3} \left[ 1 - \frac{e^{-\ell \omega_s t}}{\sqrt{1 - \xi^2}} \cdot \sin(\omega_d t + \phi) \right],$$
 (6.22)

where damped natural frequency  $\omega_{M}$  and phase shift  $\phi$  are defined as

$$\omega_d = \omega_a \sqrt{1 - \xi^2}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right).$$

The variable gains of the amplifier (i.e.,  $S_x$ ,  $T_x$ ,  $K_1$ ,  $K_y$ ) are tuned to have a desired velocity loop gain and step response characteristics as shown in Figure 6.5. When a unit step input (i.e.,  $V_x = 1 \text{ V}$ ) is applied on the amplifier input port, the maximum response of the velocity loop occurs at time  $t_y$  where the derivative of the velocity is zero (i.e.,  $d\omega(t)/dt = 0$ ). At the first overshoot, the following expression can be obtained from the time derivative of Eq. (6.22):

$$t_p = \frac{\pi}{m}$$
. (6.23)

Typical design values for a feed drive servo may be a damping ratio of  $\xi = 0.707$  and a peak time of  $t_p = 10$  ms. The natural frequency  $\omega_n$  can be estimated from Eq. (6.23) as follows:

$$w_n = \frac{x}{t_s \sqrt{1-t^2}}$$
  
= 444 rad/s = 70 Hz.

The corresponding servoparameters are identified by substituting the values of  $\xi$  and  $\omega_n$  into Eq. (6.21). It is fairly obvious that one can not demand a higher natural frequency than the maximum capacities of the motor and amplifier gains can provide.

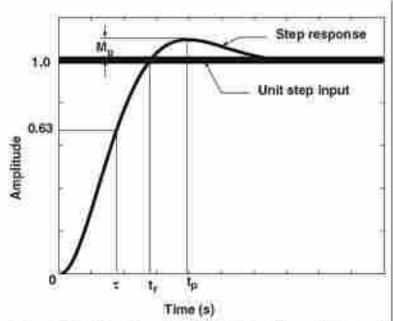


Figure 6.5: Step response of a second-order underdamped system (r = time constant,  $t_p = peak$  time,  $t_r = rise$  time,  $M_p = overshoot$ ).